

REINFORCED CONCRETE DESIGN

STEEL AND PROPERTIES

Steel Buildings and Design Criteria

$$E = \text{stress/strain}$$
$$= 29,000 \text{ ksi}$$

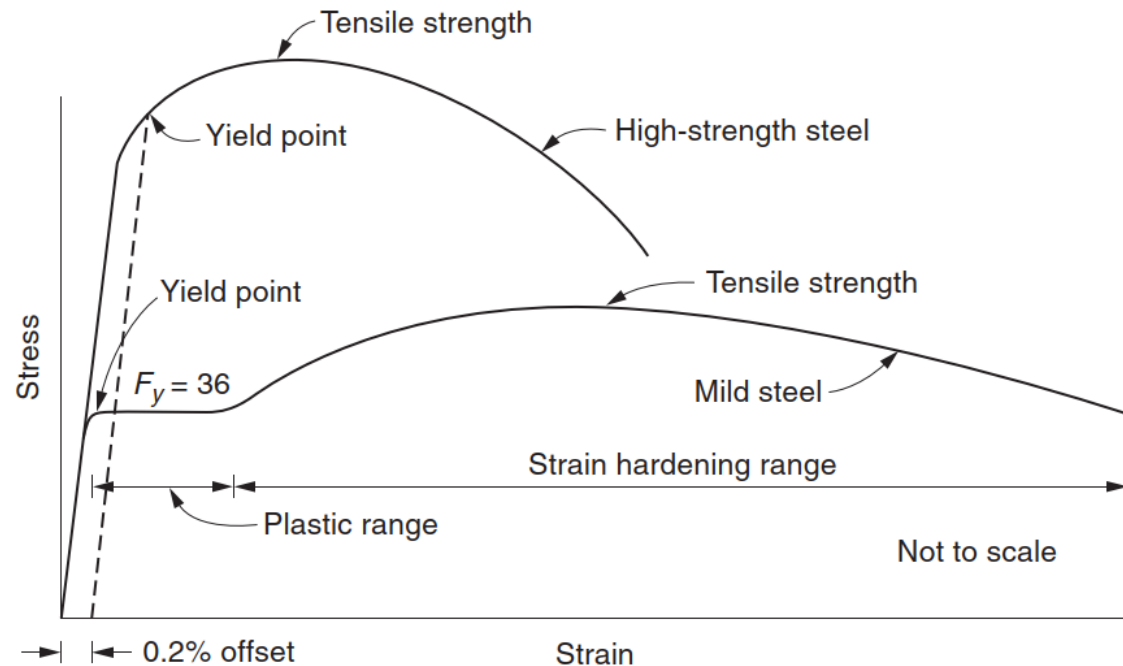


FIGURE 1.1 Stress-strain curves for steel.



W-Shapes



M-Shapes



S-Shapes



HP-Shapes



C-Shapes



L-Shapes



WT-Shapes



ST-Shapes



HSS-Shapes



Pipe

FIGURE 1.2 Standard rolled shapes.

Shape	Designation
Wide flanged beams	W
Miscellaneous beams	M
Standard beams	S
Bearing piles	HP
Standard channels	C
Miscellaneous channels	MC
Angles	L
Tees cut from W-shapes	WT
Tees cut from M-shapes	MT
Tees cut from S-shapes	ST
Rectangular hollow structural sections	HSS
Square hollow structural sections	HSS
Round hollow structural sections	HSS
Pipe	Pipe

TABLE 1.1 Rolled Steel Sections

Shape	Steel Type		
	ASTM Designation	F_y , ksi	F_u , ksi
Wide flanged beams	A992	50–65	65
Miscellaneous beams	A36	36	58–80
Standard beams	A36	36	58–80
Bearing piles	A572 Gr. 50	50	65
Standard channels	A36	36	58–80
Miscellaneous channels	A36	36	58–80
Angles	A36	36	58–80
Ts cut from W-shapes	A992	50–65	65
Ts cut from M-shapes	A36	36	58–80
Ts cut from S-shapes	A36	36	58–80
Hollow structural sections, rectangular	A500 Gr. B	46	58
Hollow structural sections, square	A500 Gr. B	46	58
Hollow structural sections, round	A500 Gr. B	42	58
Pipe	A53 Gr. B	35	60

Note: F_y = specified minimum yield stress; F_u = specified minimum tensile strength

TABLE 1.2 Type of Steel Used

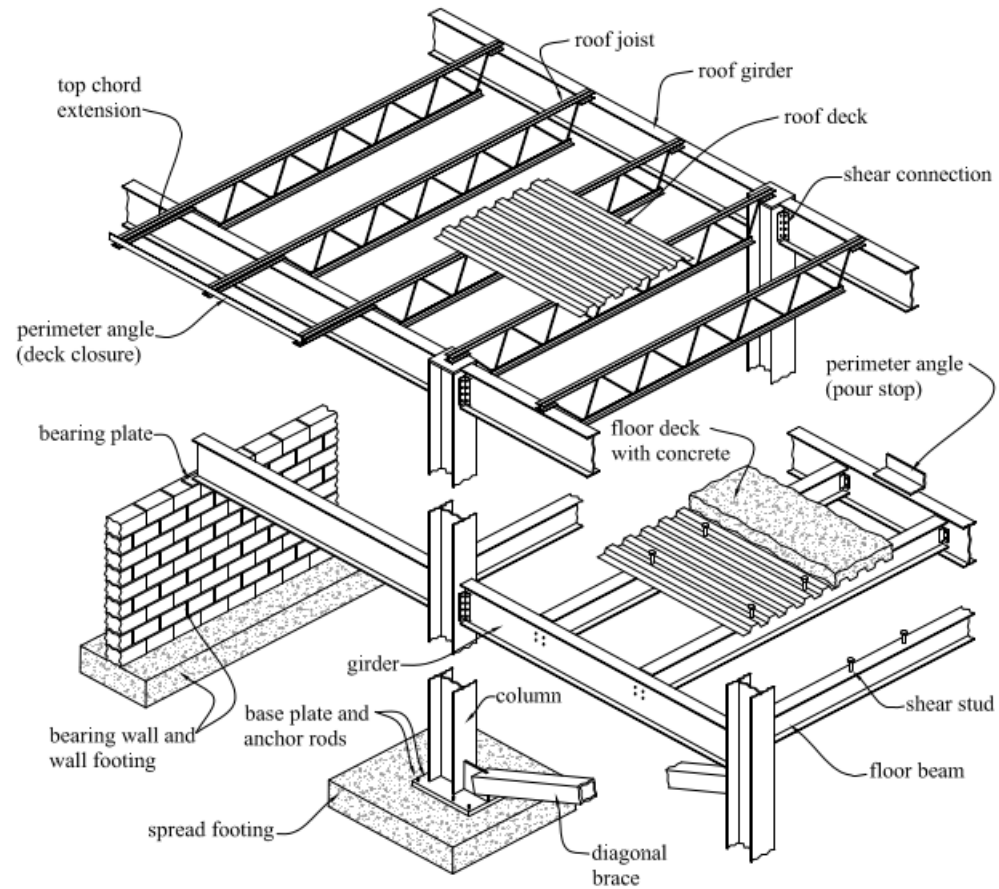


FIGURE 1-16 Typical steel building – basic structural elements (3D)

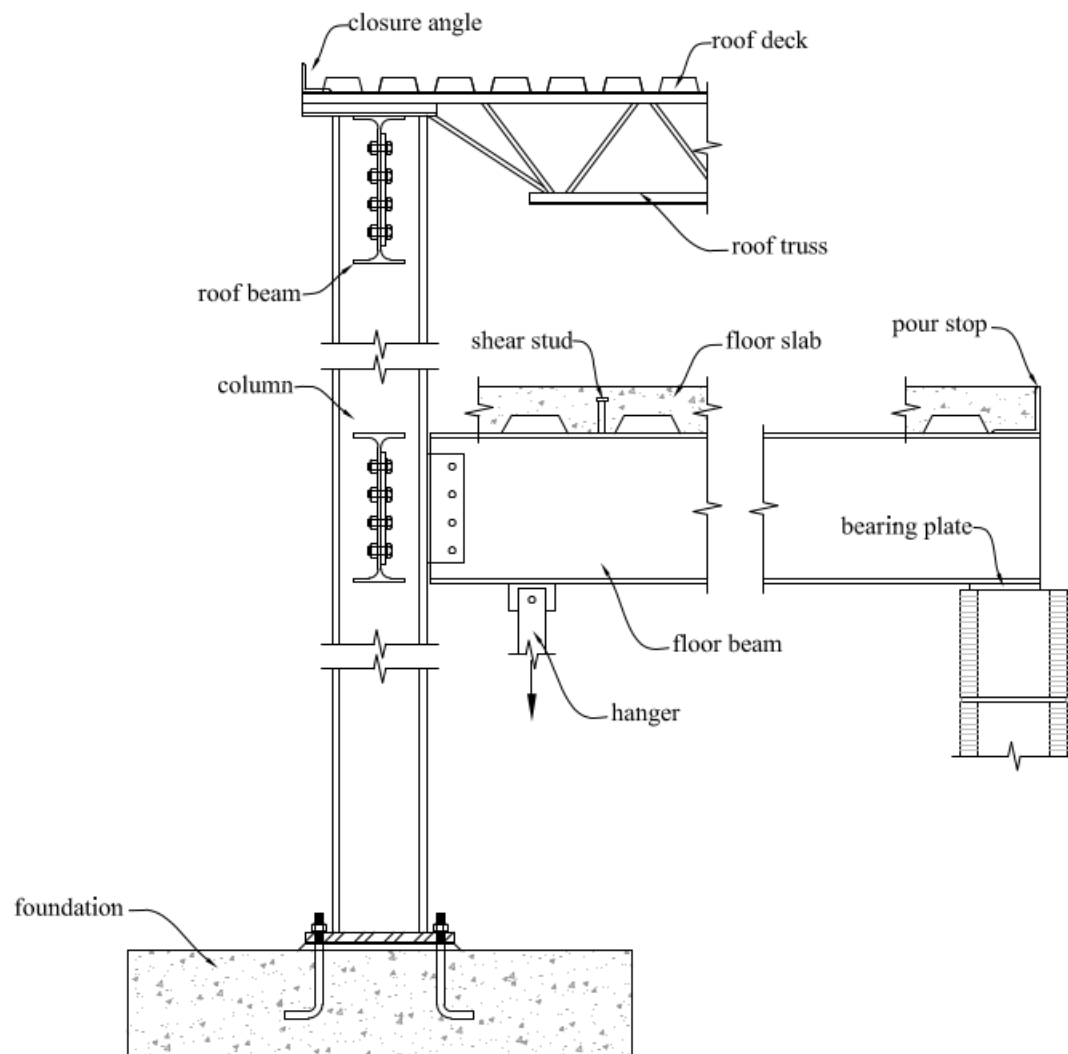


FIGURE 1-17 Typical steel building cross-section – basic structural elements

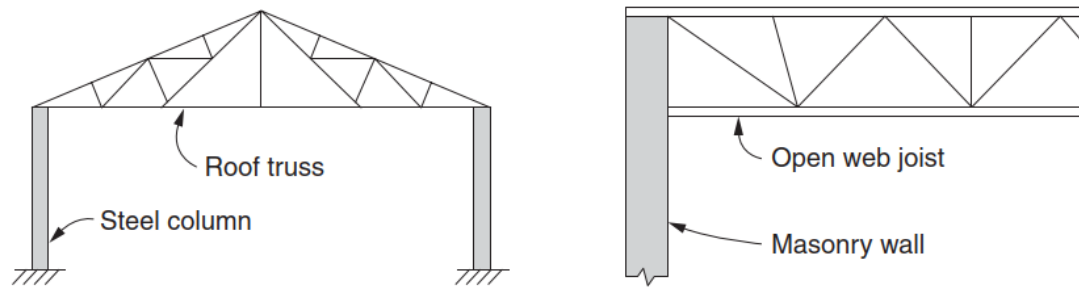


FIGURE 1.3 Steel roof construction.

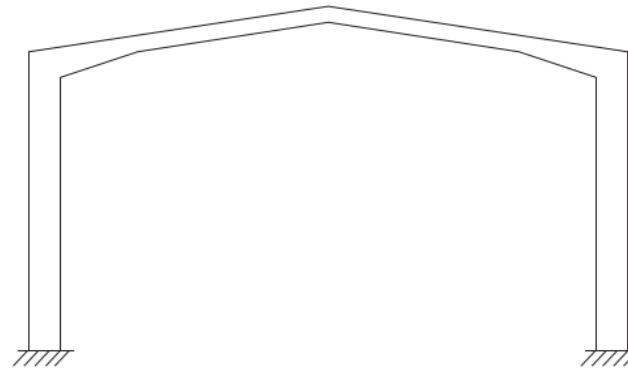


FIGURE 1.4 Single bay rigid frame.

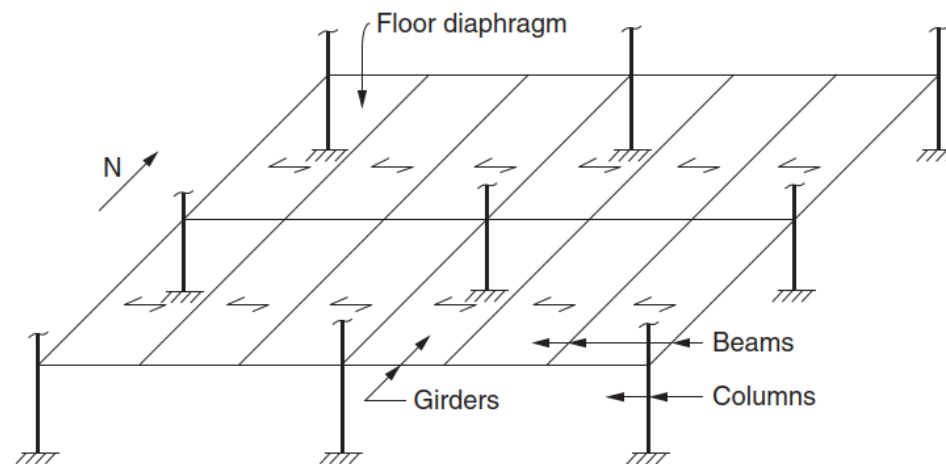


FIGURE 1.5 Framed building.

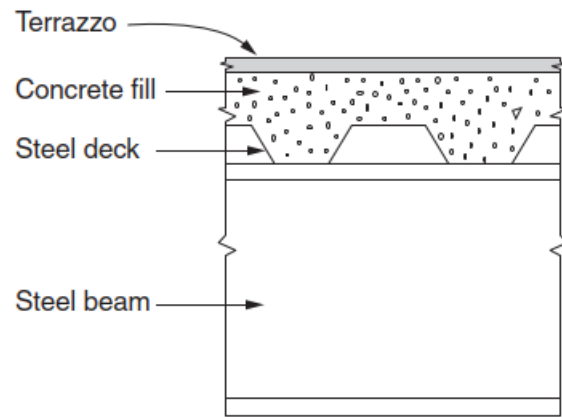


FIGURE 1.6 Beam detail.

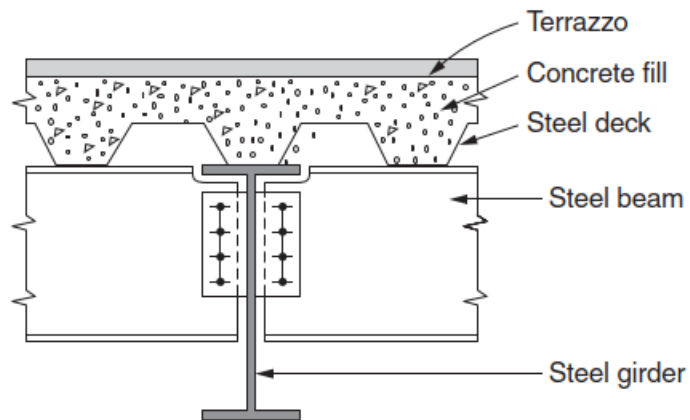


FIGURE 1.7 Girder detail.

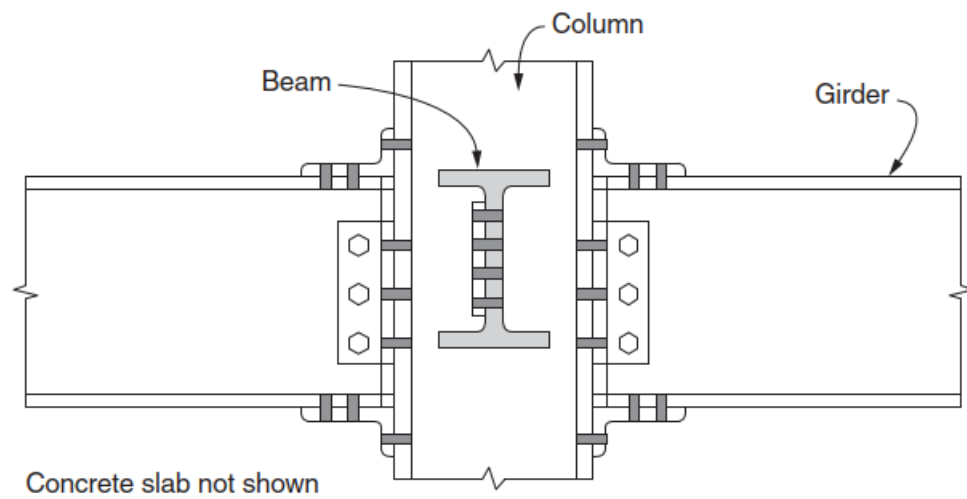


FIGURE 1.8 Column detail.

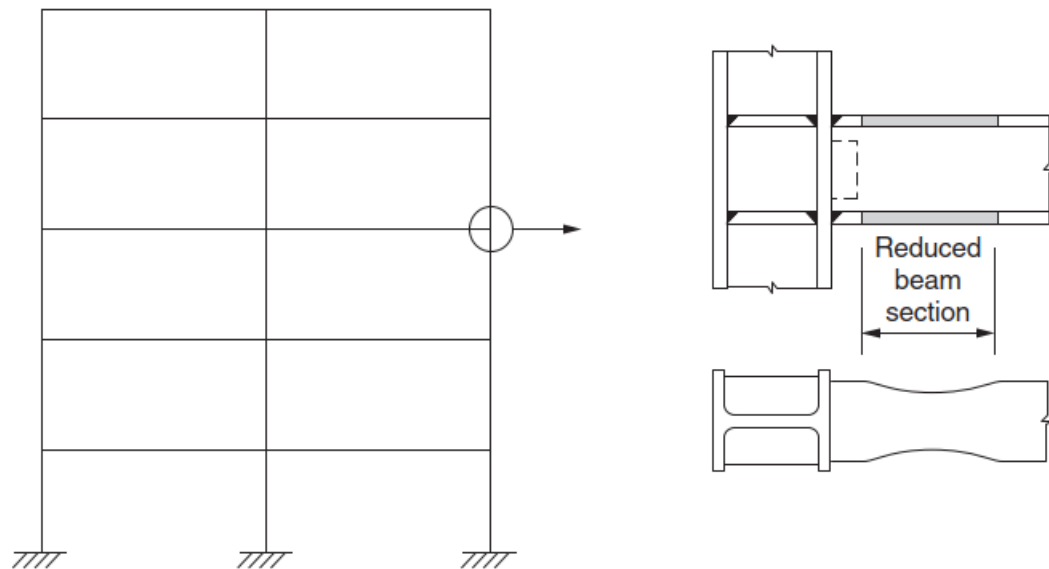


FIGURE 1.9 Moment-resisting frame.

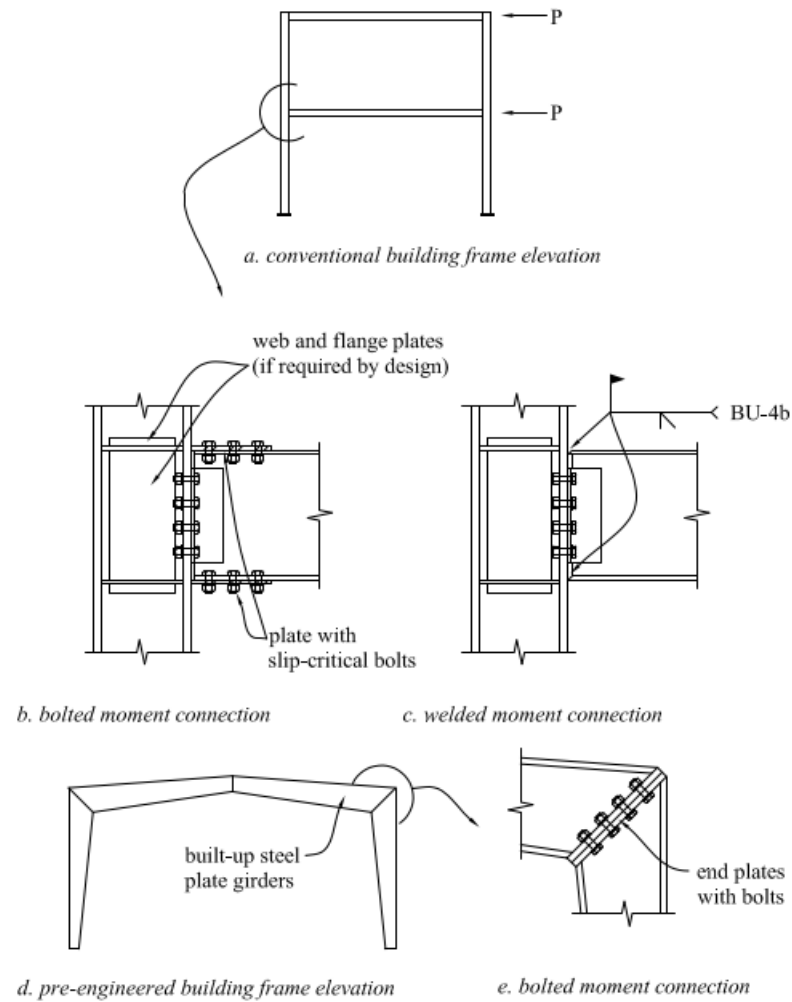


FIGURE 1-19 Typical moment resisting frames

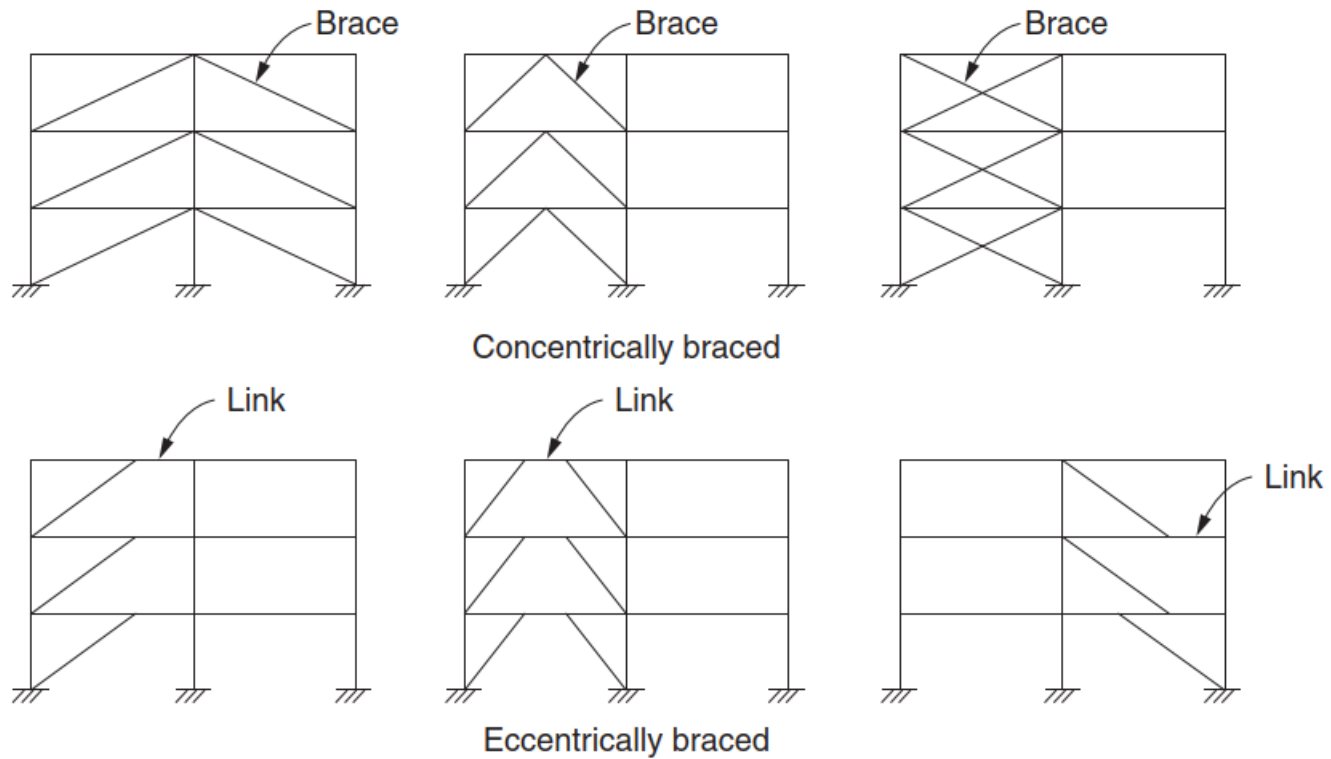
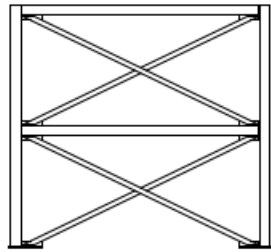
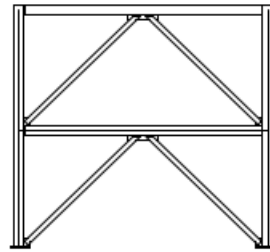


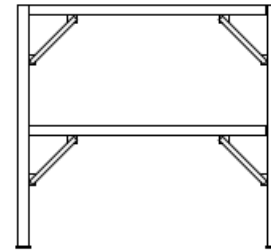
FIGURE 1.10 Braced frames.



X-Brace



Chevron or K-Brace



Knee Brace

FIGURE 1-20 Typical braced frames

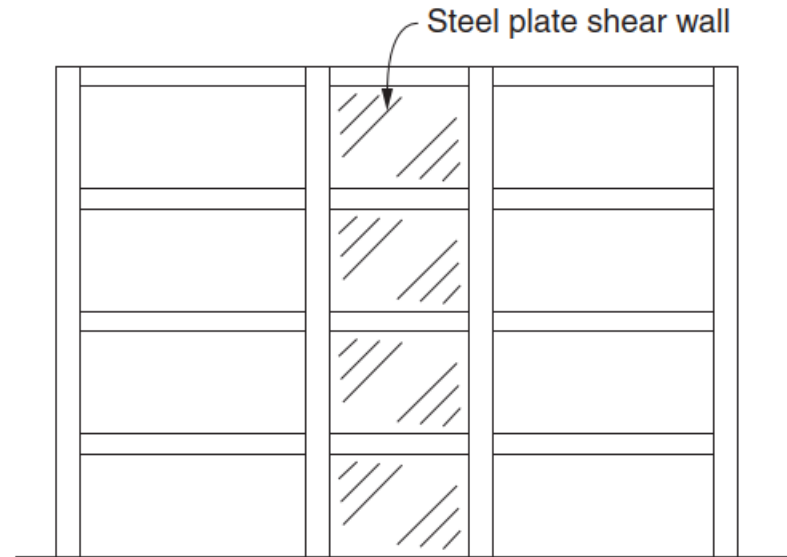


FIGURE 1.11 Steel plate shear wall building.

ASD and LRFD Concepts

$$F = F_y / \Omega$$
$$\geq f$$

where F = allowable stress

F_y = yield stress

Ω = factor of safety

f = actual stress in a member, subjected to working loads, as determined by elastic theory

$$R_u \leq \phi R_n$$

where R_u = required strength of a member subjected to strength design load combinations (LRFD)

ϕ = resistance factor

R_n = nominal strength of the member as determined by the specifications or provisions

ϕR_n = design strength

TABLE 2-2 Resistance Factors

Limit state	Resistance factor (ϕ)
Shear	1.0 or 0.9
Flexure	0.90
Compression	0.90
Tension (yielding)	0.90
Tension (rupture)	0.75

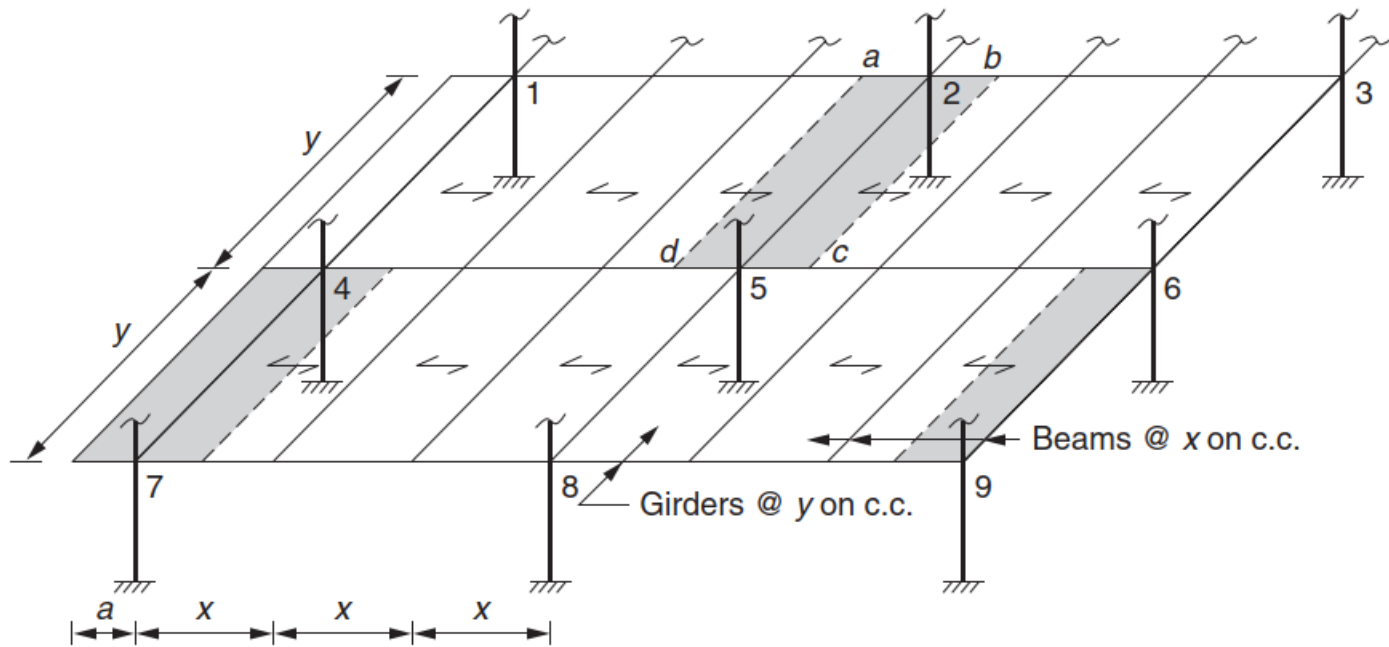


FIGURE 2.1 Beam tributary area.

Tension Members

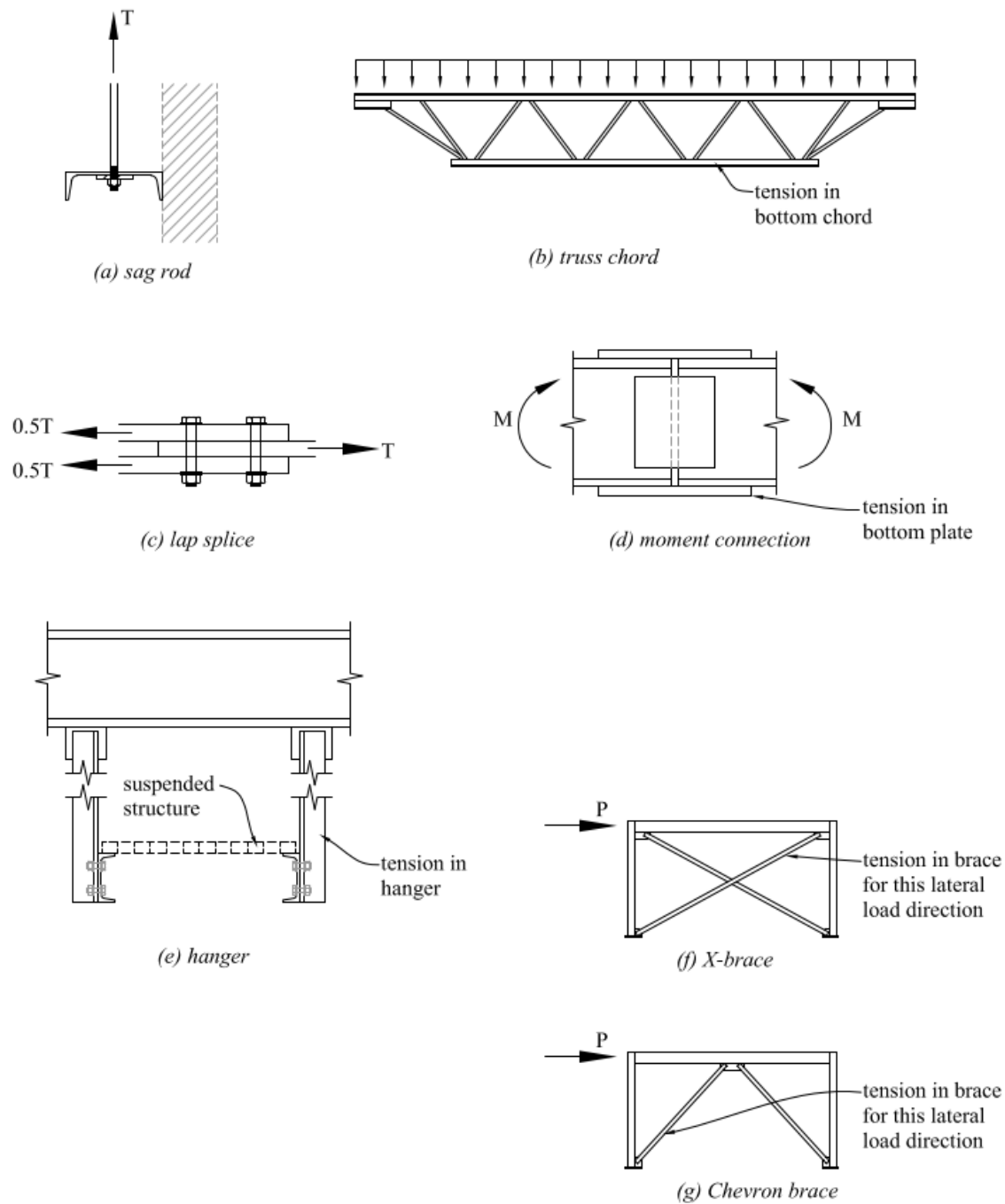


FIGURE 4-1 Common tension members

Tensile Yielding on the Gross Section:

Failure occurs in this mode by tensile yielding of the member at the gross cross-section. In the LRFD method, the ultimate tension capacity or design tensile strength, ϕP_n , for the failure mode of tensile yielding in the gross section is given as

$$\phi_t P_n = \phi_t F_y A_g, \quad (4-1a)$$

where

$$\phi_t = 0.90,$$

$$P_n = \text{Nominal tensile yield strength} = F_y A_g$$

$$F_y = \text{Minimum yield stress, and}$$

$$A_g = \text{Gross cross-sectional area of the tension member.}$$

Tensile Rupture in the Net Section:

In this failure mode, the tension member fails by tensile rupture at the net cross-section. For the LRFD method, the design tensile strength due to tensile rupture on the effective net area is given as

$$\phi_R P_n = \phi_R F_u A_e, \quad (4-2a)$$

where

$$\phi_R = 0.75,$$

$$P_n = \text{Nominal tensile strength in fracture or rupture} = F_u A_e,$$

$$F_u = \text{Minimum tensile strength, and}$$

$$A_e = \text{Effective cross-sectional area of the tension member.}$$

Effective Area, A_e :

$$A_e = A_n U,$$

where

A_n = Net cross-sectional area of the tension member, and

U = Shear lag factor.

$$A_n = A_g - \sum A_{\text{holes}},$$

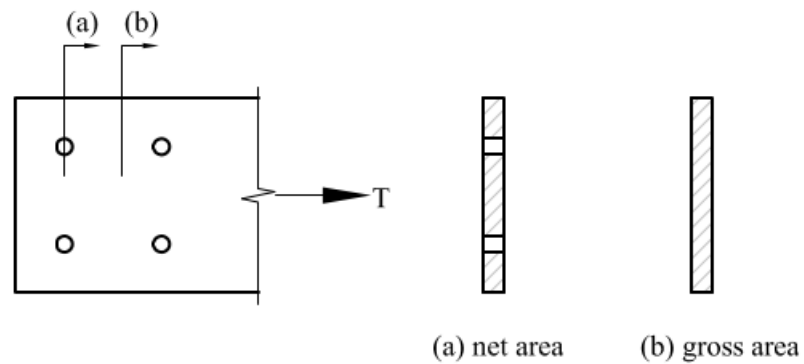


FIGURE 4-2 Tension member with in-line fasteners

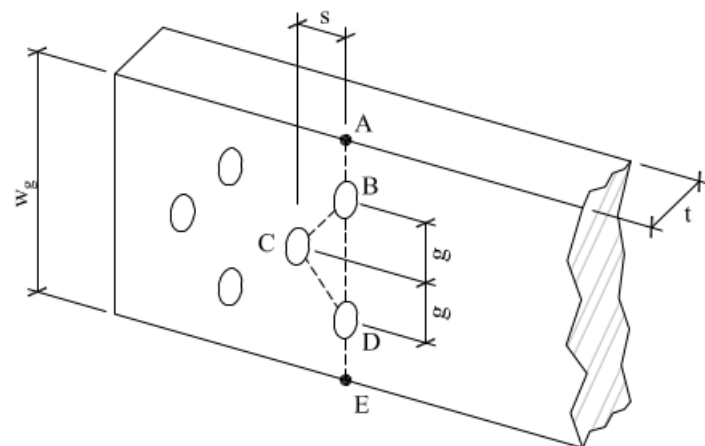


FIGURE 4-3 Tension member with diagonal fasteners

$$\frac{s^2}{4g},$$

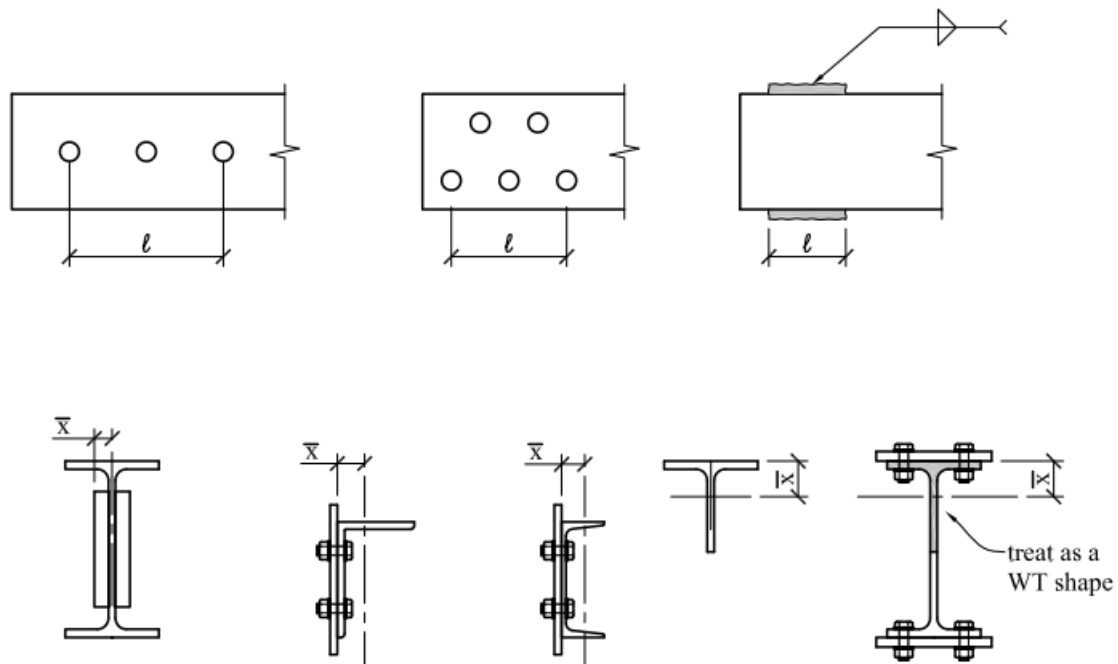
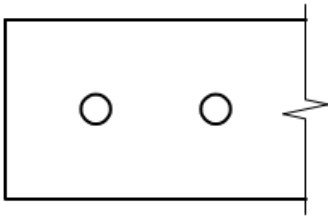
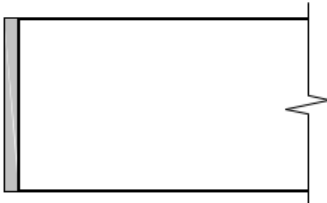
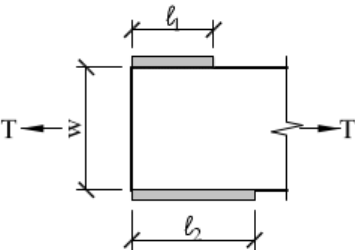
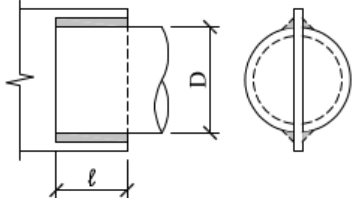
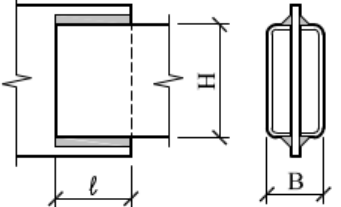
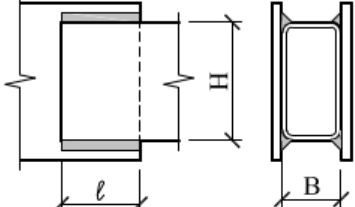
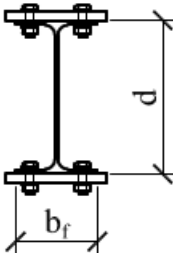
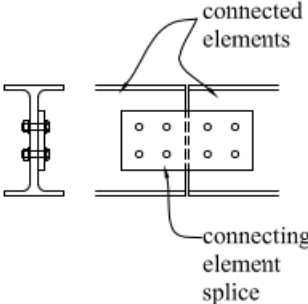
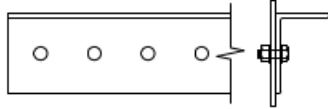
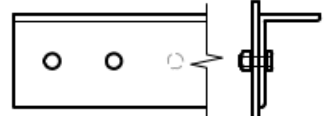


FIGURE 4-5 Determination of $\bar{x}$ and l

TABLE 4-1 Shear Lag Factor for Common Tension Member Connections

Tension member type	Description	Shear lag factor, U	Example
1a. All tension members where the axial tension load is transmitted directly to all of its component elements	All bolted*	$U = 1.0$	
1b. All tension members where the axial tension load is transmitted directly to all of its component elements	All welded	$U = 1.0$	
*2. All tension members with axial tension load transmitted to some but not all of the elements connected by bolts or a combination of long and transverse welds	Bolts or longitudinal plus transverse welds	$U = 1 - \frac{\bar{x}}{\ell}$ see Figure 4-5 for the definition of $\bar{x}$ and ℓ	

Tension member type	Description	Shear lag factor, U	Example
3. All tension members connected to some but not all the elements of the member	Transverse welds only	$U = 1.0$ $A_n =$ area of directly connected element	
4. Plates, angles, channels with welds at heels, tees, and W-shapes	Longitudinal welds only	$\frac{3\ell^2}{3\ell^2 + w^2} \left(1 - \frac{\bar{x}}{\ell} \right)$ where $\ell = \frac{\ell_1 + \ell_2}{2}$	
5. Round HSS	Single concentric gusset plate	$\ell \geq 1.3D, U = 1.0$	
		$D \leq \ell < 1.3D,$ $U = 1 - \frac{\bar{x}}{\ell}$	
		$\bar{x} = \frac{D}{\pi}$	
6a. Rectangular HSS	Single concentric gusset plate	$l \geq H, U = 1 - \frac{\bar{x}}{\ell}$	
		$\bar{x} = \frac{B^2 + 2BH}{4(B+H)}$	
6b. Rectangular HSS	Two-sided gusset plates	$l \geq H, U = 1 - \frac{\bar{x}}{\ell}$	
		$\bar{x} = \frac{B^2}{4(B+H)}$	

Tension member type	Description	Shear lag factor, U	Example
7. W , M , S , or HP , or Tees Cut from these shapes	Flange connected with three or more fasteners per line in the direction of the load	$b_f \geq \frac{2}{3}d, U = 0.90$ $b_f < \frac{2}{3}d, U = 0.85$	
	Web connected with four or more fasteners per line in the direction of the load	$U = 0.70$	
8. Single and double angles	Four or more fasteners per line in the direction of the load	$U = 0.80$	
	Two or three fasteners per line in the direction of the load	$U = 0.60$	

Source: Adapted from *AISC*, Table D3.1 [1]. Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved.

* $U \geq \text{Area of connected elements}/A_g$ (See *AISC* Specification Section D3)

U-Value for a Bolted Connection

For the bolted tension member shown in Figure 4-6, determine the shear lag factor, U ; the net area, A_n ; and the effective area, A_e .

Solution

From the section property tables in part 1 of the *AISC*, we find that for an $L5 \times 5 \times \frac{3}{8}$,

$$\bar{x} = 1.37 \text{ in.}$$

$$A_g = 3.65 \text{ in.}^2$$

Total length of the connection (i.e., distance between extreme bolts), $\ell = 9 \text{ in.}$

Shear Lag Factor:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{\ell} \\ &= 1 - \frac{1.37 \text{ in.}}{9 \text{ in.}} = 0.848 \end{aligned}$$

Alternatively, $U = 0.80$ from Table 4-1. T

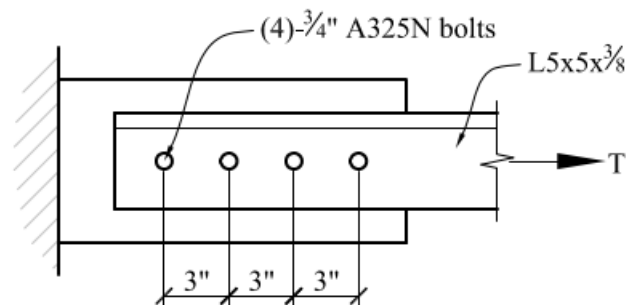


FIGURE 4-6 Detail for Example 4-1

Net Area of the Angle:

At any critical section where rupture may occur perpendicular to the tension load, there is only one hole. Therefore, the net area is

$$\begin{aligned} A_n &= A_g - \sum A_{\text{holes}} \\ &= 3.65 - (1) \left(\frac{3}{4} + \frac{1}{8} \right) (0.375) = 3.32 \text{ in.}^2 \end{aligned}$$

Effective Area:

$$\begin{aligned} A_e &= A_n U \\ &= (3.32)(0.848) = 2.82 \text{ in.}^2 \end{aligned}$$

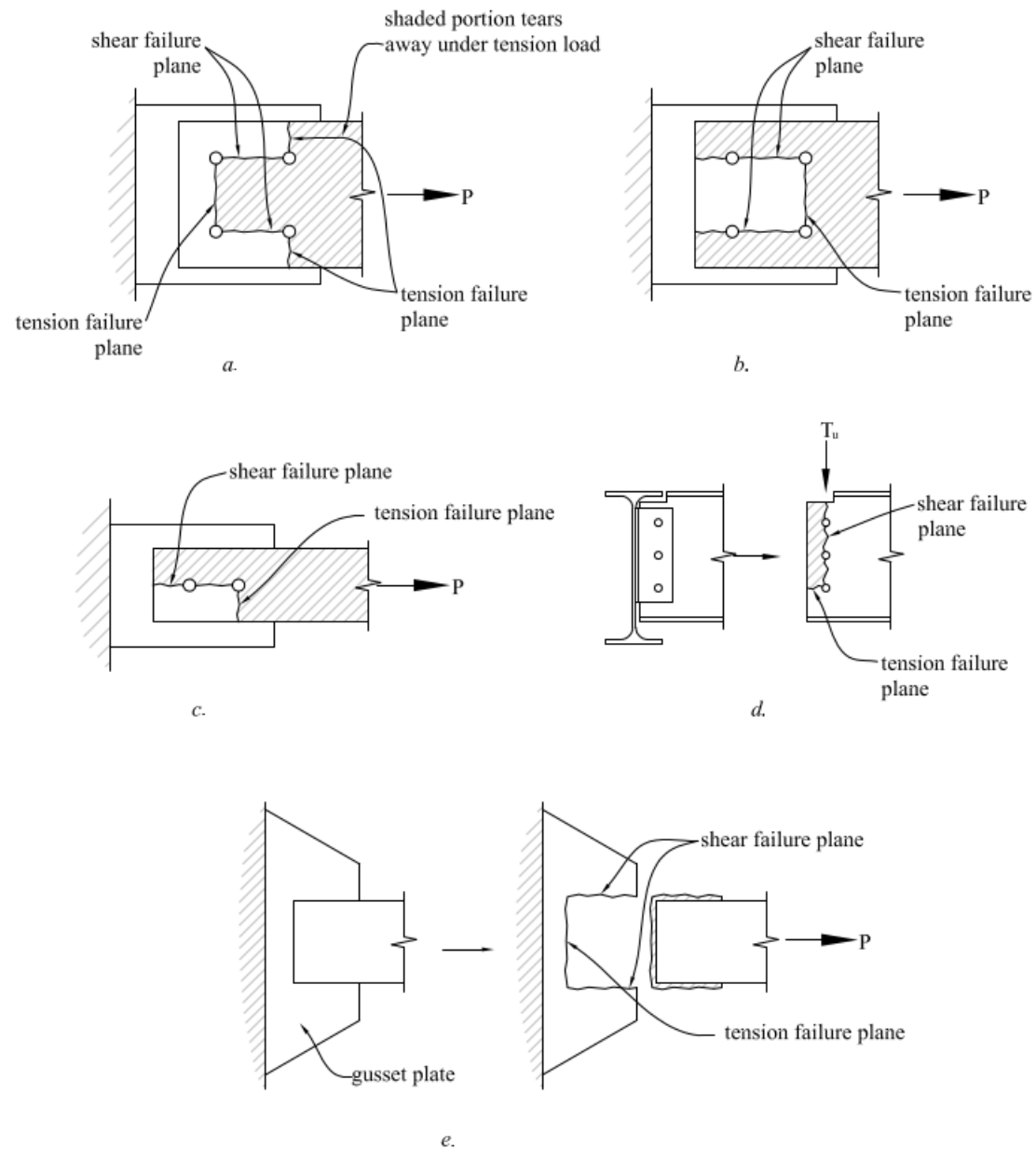


FIGURE 4-11 Block shear failure modes

EXAMPLE 4-6

Tension Member with Block Shear

For the connection shown in Example 4-4, determine if the channel and gusset plate are adequate for the applied tension load considering block shear. Assume that the width of the plate is such that block shear along the failure plane shown in Figure 4-12 controls the design of the plate.

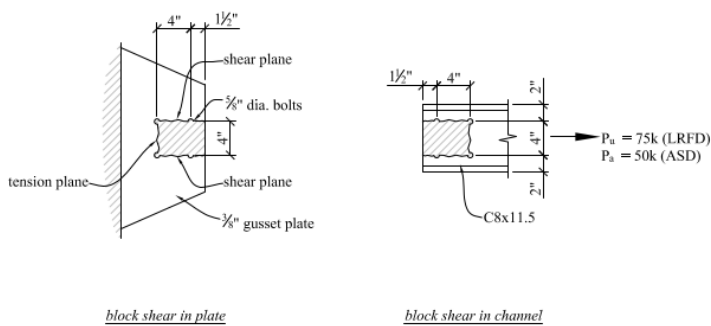


FIGURE 4-12 Detail for Example 4-6

SOLUTION

From the AISC Table 2-3:

$F_y = 36 \text{ ksi}$
 $F_u = 58 \text{ ksi to } 80 \text{ ksi (use } F_u = 58 \text{ ksi)}$

Gusset Plate Dimensions:

Shear plane:

$A_{gv} = (2 \text{ shear planes})(4 \text{ in.} + 1.5 \text{ in.})(0.375 \text{ in.}) = 4.12 \text{ in.}^2$
 $A_{nv} = A_{gv} - \sum A_{\text{holes}}$
 $= 4.12 - \left[(2 \text{ sides})(1.5 \text{ holes}) \left(\frac{5}{8} + \frac{1}{8} \right) (0.375 \text{ in.}) \right] = 3.28 \text{ in.}^2$

Tension plane:

$A_{nt} = A_{gt} - \sum A_{\text{holes}}$
 $= [(4 \text{ in.})(0.375 \text{ in.})] - \left(\frac{1}{2} \text{ hole} + \frac{1}{2} \text{ hole} \right) \left[\left(\frac{5}{8} + \frac{1}{8} \right) (0.375 \text{ in.}) \right] = 1.21 \text{ in.}^2$

Channel Dimensions:

Shear plane:

$A_{gv} = (2 \text{ shear planes})(4 \text{ in.} + 1.5 \text{ in.})(0.220 \text{ in.}) = 2.42 \text{ in.}^2$
 $A_{nv} = A_{gv} - \sum A_{\text{holes}}$
 $= 2.42 - \left[(2 \text{ shear planes})(1.5 \text{ holes}) \left(\frac{5}{8} + \frac{1}{8} \right) (0.220 \text{ in.}) \right] = 1.92 \text{ in.}^2$

Tension plane:

$A_{nt} = A_{gt} - \sum A_{\text{holes}}$
 $= [(4 \text{ in.})(0.220 \text{ in.})] - \left(\frac{1}{2} \text{ hole} + \frac{1}{2} \text{ hole} \right) \left[\left(\frac{5}{8} + \frac{1}{8} \right) (0.220 \text{ in.}) \right] = 0.715 \text{ in.}^2$
 $U_{bs} = 1.0 \text{ (tension stress is uniform)}$

LRFD:

The design block shear strength is found from equations (4-11a) for the LRFD method:

$\phi P_n = \phi (0.6 F_u A_{nv} + U_{bs} F_u A_{nt}) \leq \phi (0.6 F_y A_{gv} + U_{bs} F_u A_{nt})$
For the *Gusset Plate*, the tensile strength in block shear is
 $\phi P_n = 0.75 [(0.6)(58)(3.28) + (1.0)(58)(1.21)]$
 $\leq 0.75 [(0.6)(36)(4.12) + (1.0)(58)(1.21)]$
 $= 138 \text{ kips} \leq 119 \text{ kips.}$

The smaller value controls, so the design tension strength of the gusset plate in block shear is 119 kips, which is greater than the applied factored load of $P_u = 75 \text{ kips}$.

For the *Channel* ($t_w = 0.22 \text{ in.}$), the tensile strength in block shear is

$\phi P_n = 0.75 [(0.60)(58)(1.92) + (1.0)(58)(0.715)]$
 $\leq 0.75 [(0.60)(36)(2.42) + (1.0)(58)(0.715)]$
 $= 81.2 \text{ kips} \leq 70.3 \text{ kips.}$

The smaller value controls, so the design tension strength of the channel in block shear is 70.3 kips, which is less than the applied factored load of $P_u = 75 \text{ kips}$. Hence the channel is not adequate in block shear.

ASD:

The allowable block shear strength is found from equations (4-11b) for the ASD method:

$\frac{P_n}{\Omega} = \frac{(0.6 F_u A_{nv} + U_{bs} F_u A_{nt})}{\Omega} \leq \frac{(0.6 F_y A_{gv} + U_{bs} F_u A_{nt})}{\Omega}$
For the *Gusset Plate*, the allowable tensile strength in block shear is
 $\frac{P_n}{\Omega} = \frac{(0.6)(58)(3.28) + (1.0)(58)(1.21)}{2.0} \leq \frac{(0.6)(36)(4.12) + (1.0)(58)(1.21)}{2.0}$
 $= 92.1 \text{ kips} \leq 79.6 \text{ kips.}$

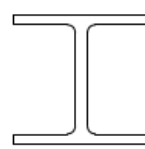
The smaller value controls, so the available strength of the gusset plate in block shear is 79.6 kips, which is greater than the applied load of $P_a = 50 \text{ kips}$.

For the *Channel* ($t_w = 0.22 \text{ in.}$), the allowable tensile strength in block shear is

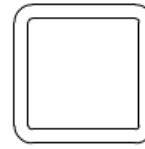
$\frac{P_n}{\Omega} = \frac{(0.6)(58)(1.92) + (1.0)(58)(0.715)}{2.0} \leq \frac{(0.6)(36)(2.42) + (1.0)(58)(0.715)}{2.0}$
 $= 54.1 \text{ kips} \leq 46.9 \text{ kips.}$

The smaller value controls, so the allowable tensile strength of the channel in block shear is 46.9 kips, which is less than the applied total service or unfactored load of $P_a = 50 \text{ kips}$. Hence the channel is not adequate in block shear.

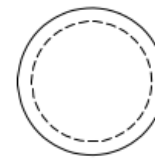
Compression Members



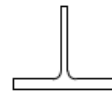
W-shape



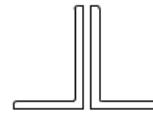
square HSS



round HSS



WT

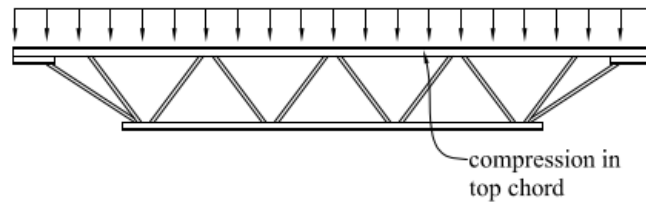


double angle

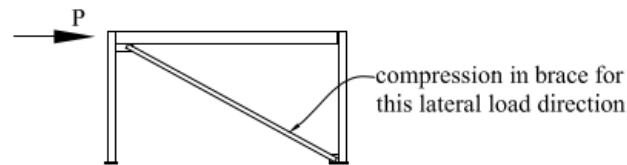


single angle

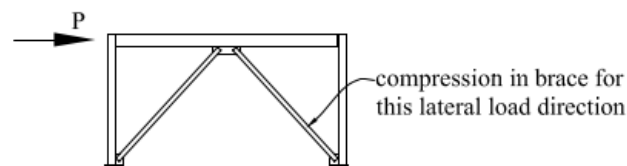
(i) Structural steel sections used as compression members



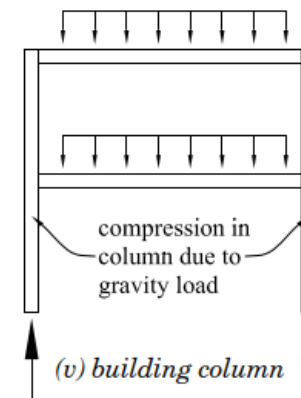
(ii) truss chord



(iii) diagonal brace



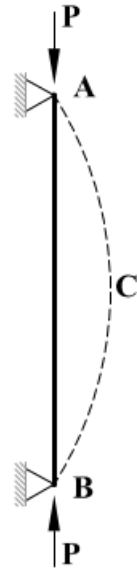
(iv) Chevron brace



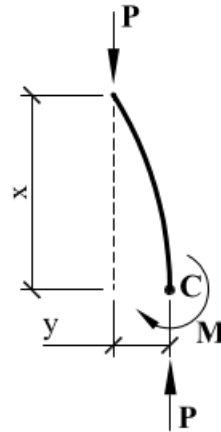
(v) building column

FIGURE 5-1a Basic types of compression members

EULER CRITICAL BUCKLING LOAD

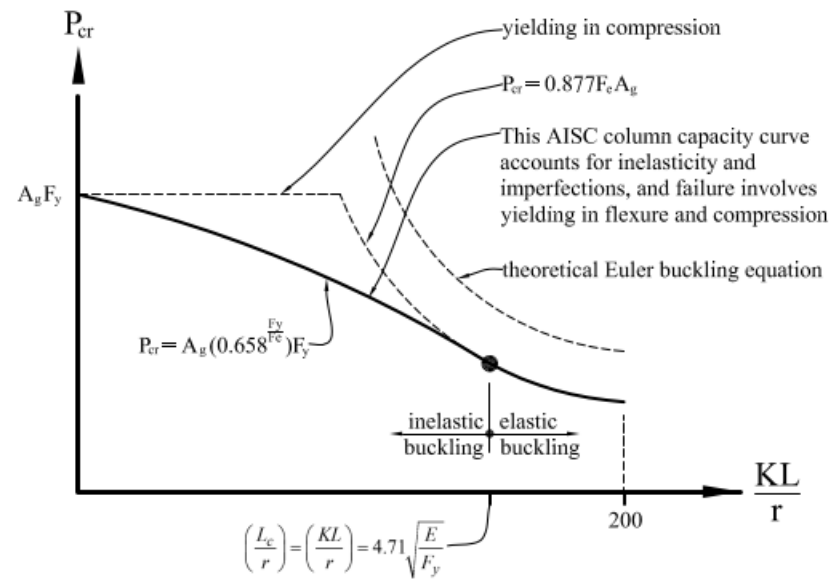
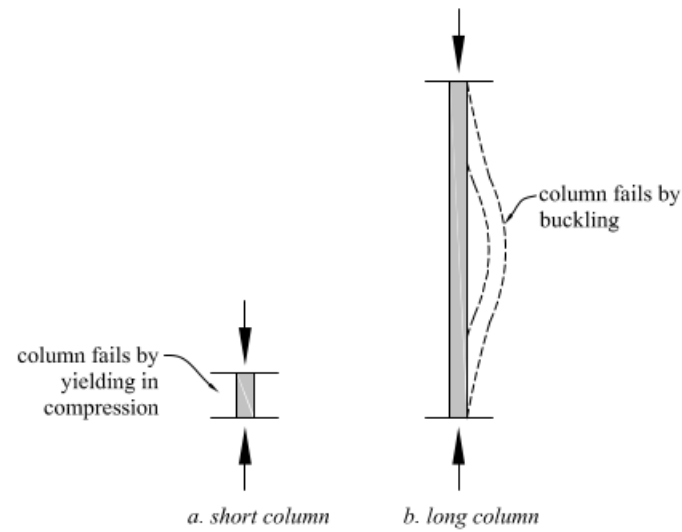


(a) Pin ended column subject to axial compression load



(b) Free body diagram

FIGURE 5-2a-b Pin ended column



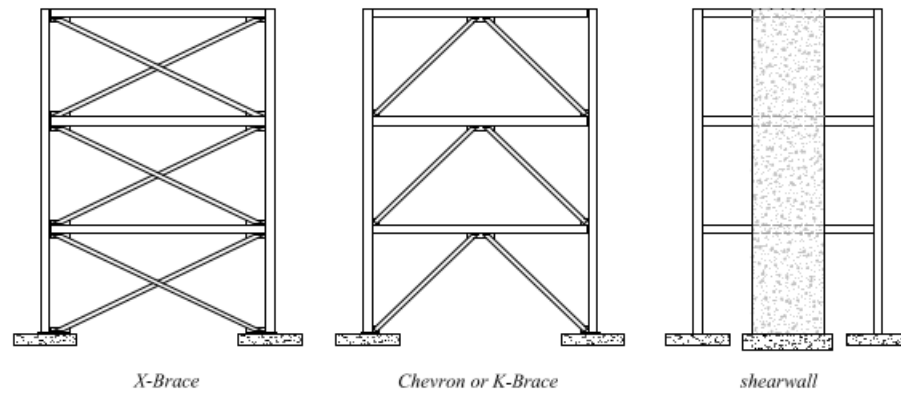
c. column capacity curve

FIGURE 5-3 Column failure modes and capacity curves

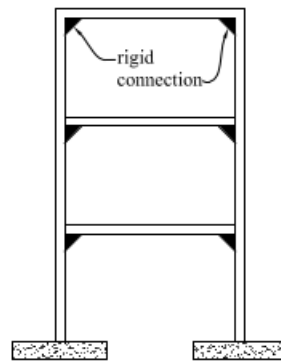
	(a)	(b)	(c)	(d)	(e)	(f)
Buckling modes						
Theoretical K value	0.50	0.70	1.0	1.0	2.0	2.0
Recommended design K when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.1	2.4
End condition legend	 Rotation fixed, translation fixed  Rotation free, translation fixed  Rotation fixed, translation free  Rotation free, translation free					

Adapted from AISCM Table C-A-7.1 [1]. Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved.

FIGURE 5-4 Effective length factors, K , for idealized support conditions



a) braced frames



b) unbraced frames

FIGURE 5-5a-b Braced and unbraced frames

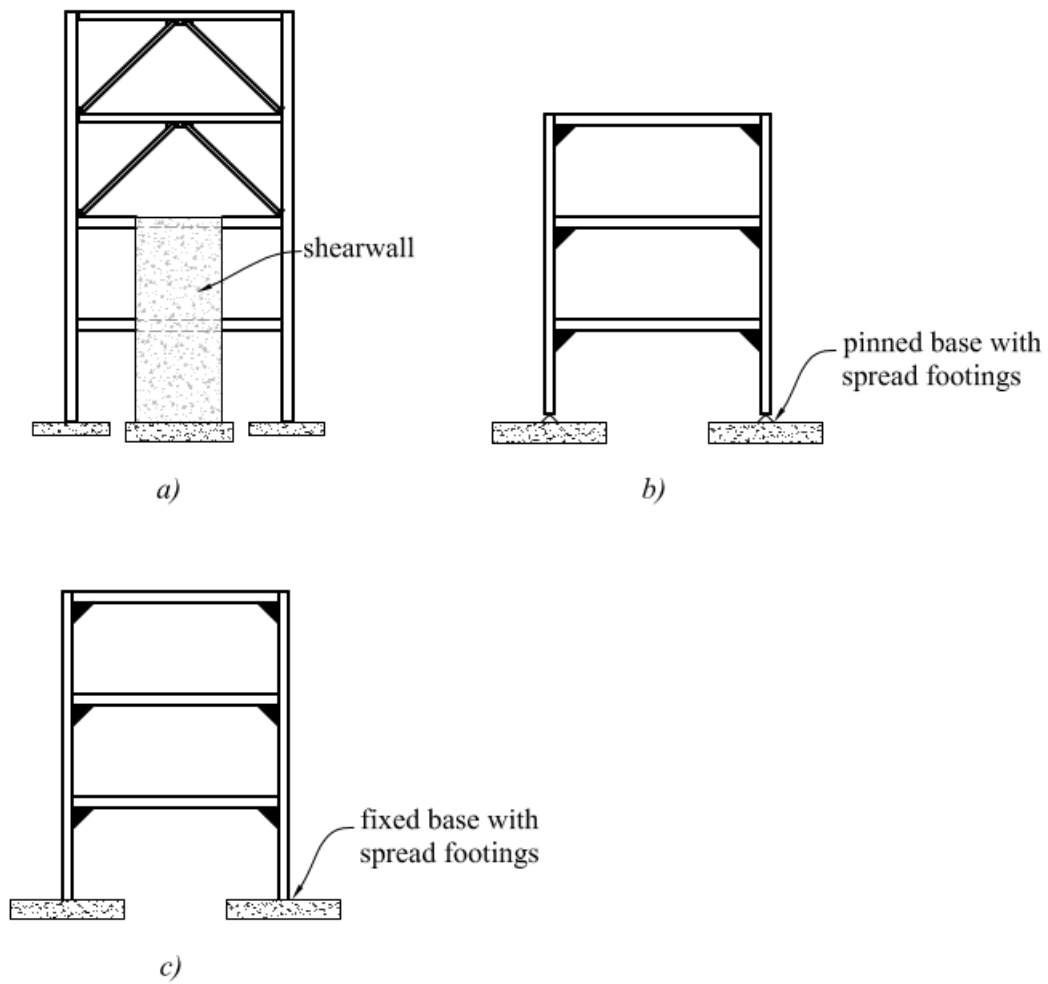


FIGURE 5-6 Details for Example 5-1

COMPRESSION MEMBER STRENGTH

The nominal axial compression strength is given as

$$P_n = F_{cr} A_g,$$

Where,

P_n = Nominal compressive strength ($= F_{cr} A_g$), kips,

F_{cr} = Flexural buckling stress (see below), ksi

A_g = Gross cross-sectional area of the column, in.²

For the LRFD method, the design axial compressive strength is given as

$$\phi_c P_n = \phi_c F_{cr} A_g,$$

For the ASD method, the allowable axial compressive load is given as

$$P_a = \frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{\Omega_c}$$

Where,

ϕ_c = strength reduction factor for compression = 0.90,

Ω_c = factor of safety for compression = 1.67

LOCAL BUCKLING OF COMPRESSION MEMBERS

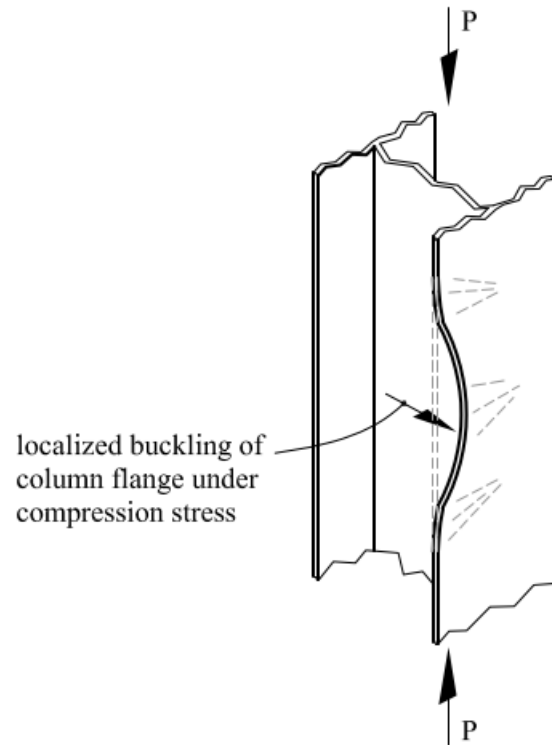


FIGURE 5-7 Local buckling of column

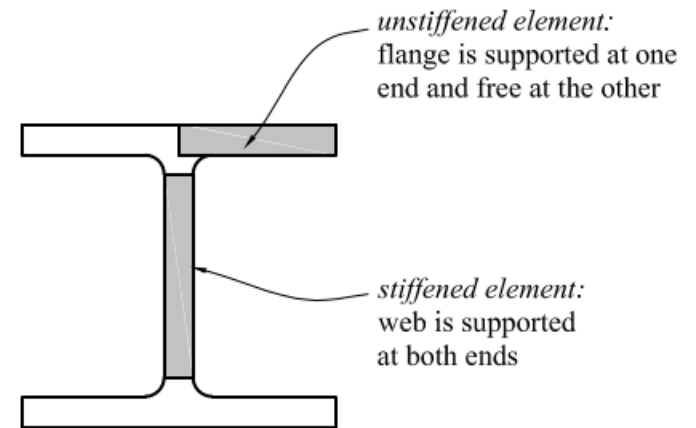
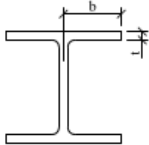
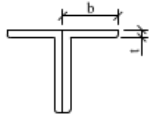
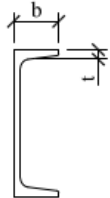
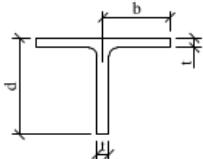
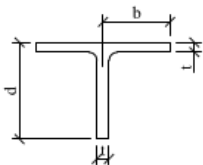
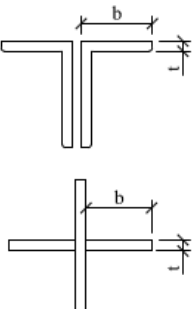
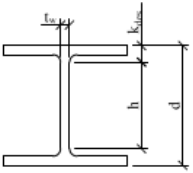
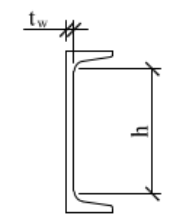
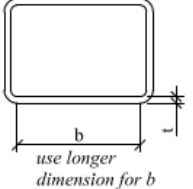
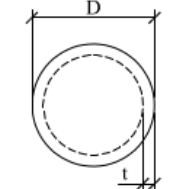


FIGURE 5-8 Stiffened and unstiffened elements

TABLE 5-1 Limiting Width-to-Thickness Ratios for Compression Elements Subject to Axial Compression*

Description		Limiting width–thickness ratio		Details
		λ (actual width-to-thickness ratio of the element)	λ_r ($\lambda \leq \lambda_r$ = non-slender; $\lambda > \lambda_r$ = slender)	
Unstiffened	Flanges of I-shaped sections (Case 1)	$\frac{b}{t}$	$0.56 \sqrt{\frac{E}{F_y}}$	
	Outstanding legs of double angles in continuous contact (i.e., with no separators) (Case 1)			
	Flanges of C-shapes (Case 1)			
	Flanges of WT-shapes (Case 1)	$\frac{b}{t}$	$0.56 \sqrt{\frac{E}{F_y}}$	
	Stems of WT-shapes (Case 4)	$\frac{d}{t}$	$0.75 \sqrt{\frac{E}{F_y}}$	
	Outstanding legs of single angles and double angles not in continuous contact (i.e., with separators), and cruciform shapes (Case 3)	$\frac{b}{t}$	$0.45 \sqrt{\frac{E}{F_y}}$	

Description		Limiting width–thickness ratio		Details
		λ (actual width-to-thickness ratio of the element)	λ_r ($\lambda \leq \lambda_r$ = non-slender; $\lambda > \lambda_r$ = slender)	
Stiffened	Webs of I-shaped sections (Case 5)	$\frac{h}{t_w}$	$1.49 \sqrt{\frac{E}{F_y}}$	
	Webs of C-shapes (Case 5)			
	Built-up box sections (Case 8) b = clear distance between walls of box section; t = wall thickness	$\frac{b}{t}$	$1.49 \sqrt{\frac{E}{F_y}}$	
	Square or rectangular HSS (Case 6)	$\frac{b}{t}$	$1.40 \sqrt{\frac{E}{F_y}}$	
	Round HSS or pipes (Case 9)	$\frac{D}{t}$	$\frac{D}{t} \leq 0.11 \left(\frac{E}{F_y} \right)$	

* Adapted from AISC Table B4.1a [1]. Copyright © American Institute of Steel Construction. Reprinted with permission, All rights reserved.

In Table 5-1, λ is the actual width-to-thickness ratio of the element. For wide flange sections, λ for the flange and web, respectively, are calculated as follows:

Flange:

$$\lambda = \frac{b}{t} = \frac{\left(\frac{1}{2}\right)b_f}{t_f} = \frac{b_f}{2t_f}$$

Web:

$$\lambda = \frac{h}{t_w} = \frac{d - 2k_{des}}{t_w}$$

EXAMPLE 5-4

Analysis of HSS Column with Slender Elements

Calculate the design axial compressive strength, $\phi_c P_n$, of an HSS $12 \times 8 \times 3/16$ column, 28 ft. long, and pinned at both ends. Use ASTM A500 Grade C steel. Also, using the ASD method, calculate the allowable axial compression load, P_a .

Solution

The unbraced lengths are

$$L_x = L_y = 28 \text{ ft.}$$

The effective length factor is $K = 1.0$ (Figure 5-4)

$$F_y = 50 \text{ ksi}$$

For HSS $12 \times 8 \times 3/16$, the section properties from Part 1 of the AISCM are

$$A_g = 6.76 \text{ in.}^2$$

$t = 0.174 \text{ in.}$ (use only the decimal values for design)

$$b - 3t = 8 - 3(0.174) = 7.48 \text{ in.}$$

$$h - 3t = 12 - 3(0.174) = 11.48 \text{ in.}$$

$$\lambda_{(\text{short leg})} = \frac{b - 3t}{t} = \frac{7.48}{0.174} = 43$$

$$\lambda_{(\text{long leg})} = \frac{h - 3t}{t} = \frac{11.48}{0.174} = 66$$

$$r_x = 4.56 \text{ in.}$$

$$r_y = 3.35 \text{ in.}$$

Compute the slenderness ratio about the x - x and y - y axes and use the larger of the two values (i.e., the larger of $\frac{KL_x}{r_x}$ or $\frac{KL_y}{r_y}$).

Since the unbraced length is the same about both axes (i.e., $L_x = L_y$), therefore, the smaller radius of gyration will control.

$$\frac{L_{cy}}{r_y} = \frac{KL}{r} = \frac{(1.0)(28 \text{ ft.})(12)}{3.35} = 100.3 < 200 \quad \text{OK.}$$

Check the slenderness criteria for compression elements (i.e., check for local buckling):

The limiting width to thickness ratios from Table 5-1 are as follows:

$$\lambda_r \leq 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29,000}{50}} = 33.7$$

$$\lambda \text{ (short leg)} = 43 > \lambda_r = 33.7$$

$$\lambda \text{ (long leg)} = 66 > \lambda_r = 33.7$$

The actual width-to-thickness ratios, λ , for both short and long legs are greater than the limiting width-to-thickness ratio, λ_r , therefore both the short and long legs of this HSS column are slender and therefore susceptible to local buckling. Hence, the local buckling failure mode must therefore be checked.

Determine the flexural buckling stress, F_{cr} :

The maximum slenderness ratio for inelastic buckling is

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000}{50}} = 113.4$$

Since $\frac{KL}{r} = 100.3 < 113.4$, this implies that the failure mode is by inelastic flexural buckling; therefore, use equation (5-14) to determine F_{cr} .

The Euler critical buckling stress is

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29,000)}{(100.3)^2} = 28.45 \text{ ksi}$$

The flexural buckling stress is

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}} \right] F_y = \left[0.658^{\frac{50}{28.45}} \right] (50) = 24 \text{ ksi}$$

Calculate the reduction in effective area due to local buckling:

From Table 5-2 (see also AISC Specification Table E7.1), the effective width imperfection adjustment factors for walls of HSS sections are obtained as

$$c_1 = 0.2$$

$$c_2 = 1.38$$

Short legs:

The elastic local buckling stress for the short legs is

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda} \right)^2 F_y = \left(1.38 \frac{33.7}{43} \right)^2 (50 \text{ ksi}) = 58.5 \text{ ksi}$$

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 33.7 \sqrt{\frac{50}{24}} = 48.6$$

Since $\lambda \text{ (short leg)} = 43 < 48.6$, therefore, use equation (5-19a) to obtain $b_e = b = 7.48 \text{ in.}$

$$\begin{aligned} \text{The reduction in effective area for the short legs of the HSS} &= (2 \text{ short legs})(b - b_e)(t) \\ &= (2)(7.48 - 7.48)(0.174) = 0 \end{aligned}$$

Long legs:

The elastic local buckling stress for the long legs is

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda} \right)^2 F_y = \left(1.38 \frac{33.7}{66} \right)^2 (50 \text{ ksi}) = 24.8 \text{ ksi}$$

$$b_{(\text{long leg})} = 11.48 \text{ in.}$$

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 33.7 \sqrt{\frac{50}{24}} = 48.6 < \lambda_{(\text{long leg})} = 66,$$

Since $\lambda_{(\text{long leg})} = 66 > \lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 48.6$, use equation (5-19b) to obtain b_e .

$$\text{Therefore, } b_e = b \left(1 - c_1 \sqrt{\frac{F_{el}}{F_{cr}}} \right) \sqrt{\frac{F_{el}}{F_{cr}}} = 11.48 \left(1 - 0.2 \sqrt{\frac{24.8}{24}} \right) \sqrt{\frac{24.8}{24}} = 9.3 \text{ in.}$$

$$\begin{aligned} \text{The reduction in effective area for the long legs of the HSS} &= (2 \text{ long legs})(b - b_e)(t) \\ &= (2)(11.48 - 9.3)(0.174) \\ &= 0.76 \text{ in.}^2 \end{aligned}$$

The effective area, $A_e = A_g - \text{reduction in effective area for the short legs} - \text{reduction in effective area for the long legs}$

$$\text{Therefore, } A_e = 6.76 \text{ in.}^2 - 0.0 \text{ in.}^2 - 0.76 \text{ in.}^2 = 6.0 \text{ in.}^2$$

$$\text{The nominal axial load capacity of the column, } P_n = F_{cr} A_e = (24 \text{ ksi})(6.0 \text{ in.}^2) = 144 \text{ kips}$$

LRFD method:

The design axial compression load capacity for the LRFD method is

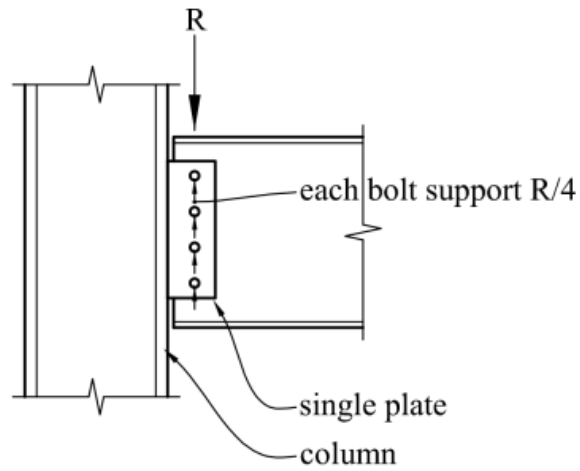
$$\phi_c P_n = (0.9)(144 \text{ kips}) = 130 \text{ kips.}$$

Compare the value obtained above to the value of $\phi_c P_n = 129 \text{ kips}$ from the column load tables, *AISC* Table 4-3, we find the results to be essentially the same.

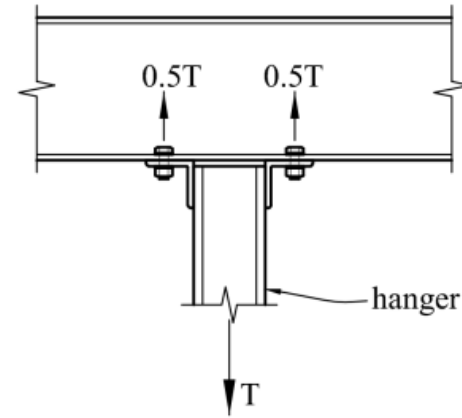
ASD method:

$$\text{The allowable axial compression load for the ASD method is } P_a = \frac{P_n}{\Omega_c} = \frac{144}{1.67} = 86 \text{ kips.}$$

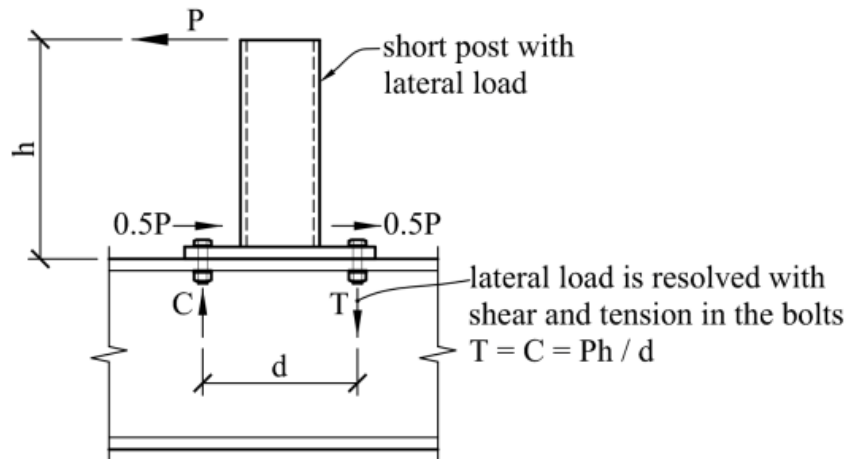
Compare to the value of $P_a = 86.1 \text{ kips}$ obtained from *AISC* Table 4-3, and the results are found to be essentially the same.



a) Bolts in Shear

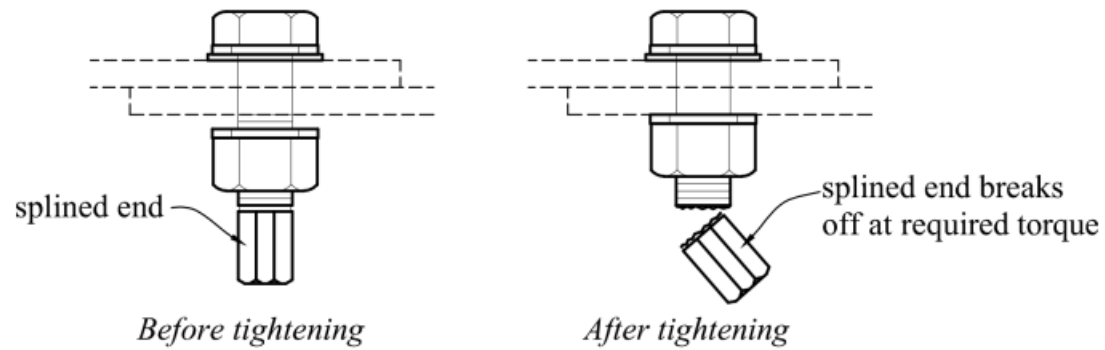


b) Bolts in Tension

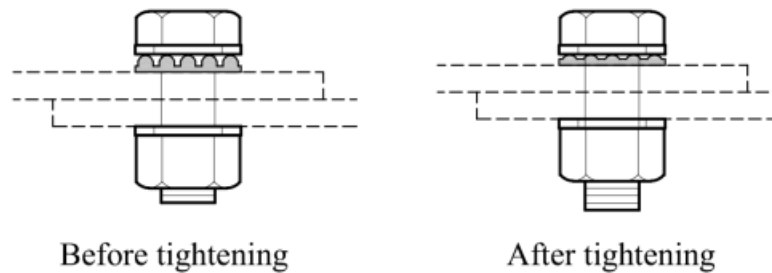


c) Bolts in Shear and Tension

FIGURE 9-1 Connections types using bolts



a) Twist-off-type tension-control bolts



b) Direct-Tension-Indicator washer

FIGURE 9-2 Bolts types based on tightening

FAILURE MODES AND STRENGTH OF BOLTS

The failure modes of bolted connections (see Figure 9-5), include:

1. Shear failure of the bolts
2. Tension failure of the bolts
3. Bearing failure of the connected elements (i.e., the main material and the connection material) that the bolts bear against. In this failure mode, the bolt deforms the edge of the material that it bears against. If the deformation at the bolt hole is a design consideration (i.e., the deformation is limited), the strength of the bolt in bearing is smaller compared to when the deformation at a bolt hole is not limited and therefore not a design consideration.

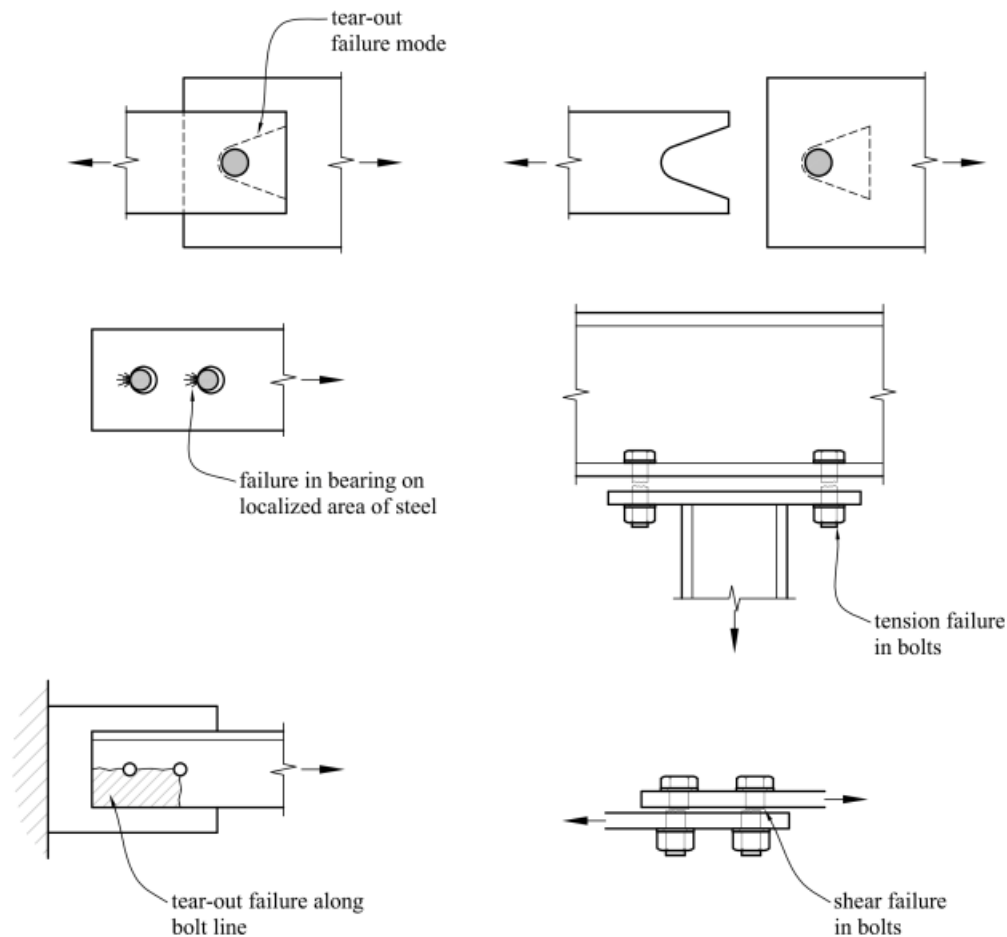


FIGURE 9-5 Bolt failure modes

The design shear and tension strengths of a snug-tight or pretensioned bolt are calculated as follows:

$$\phi R_n = \phi F_n A_b, \quad \text{for LRFD} \quad (9-1a)$$

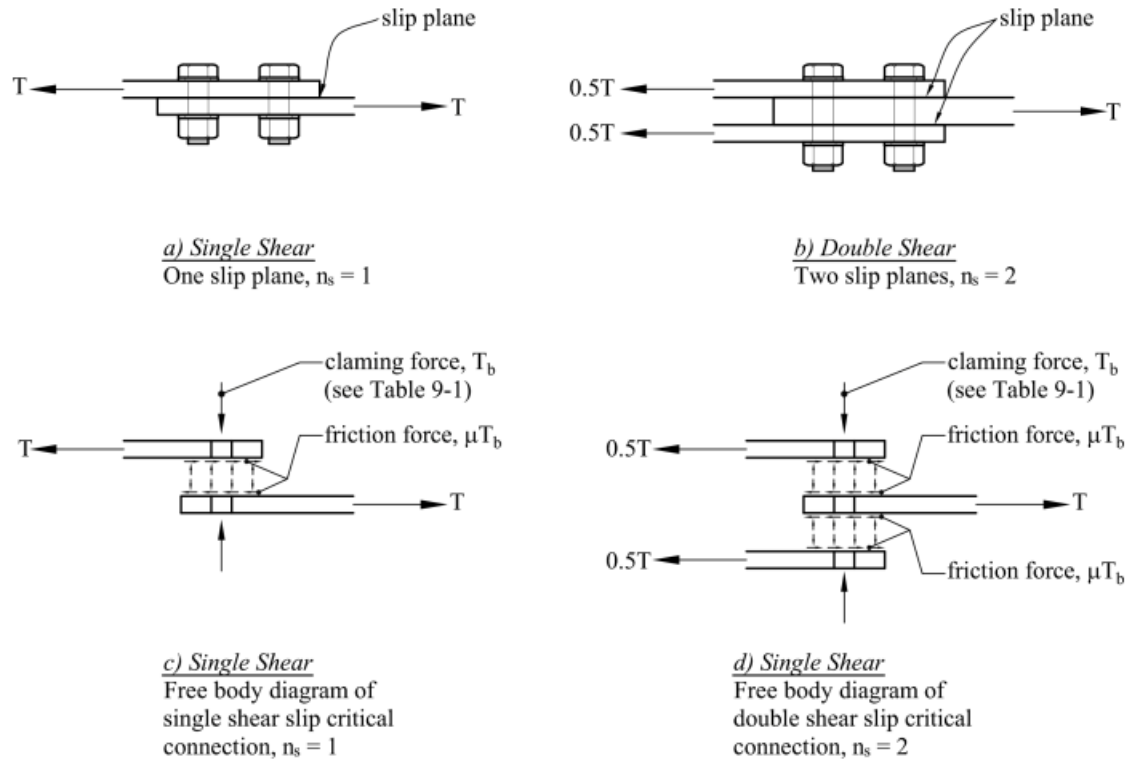


FIGURE 9-6 Bolts loaded in single and double shear

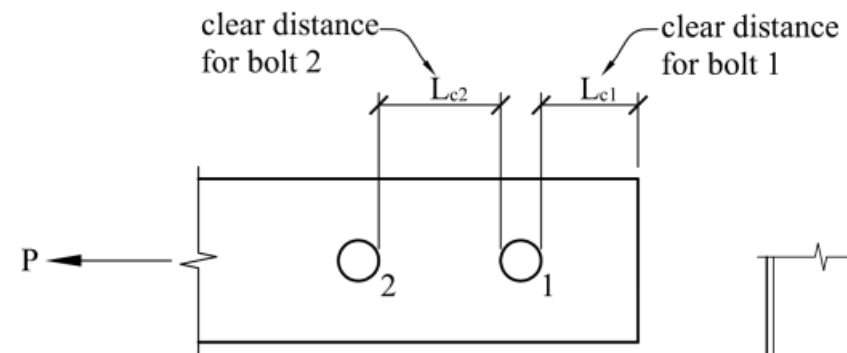
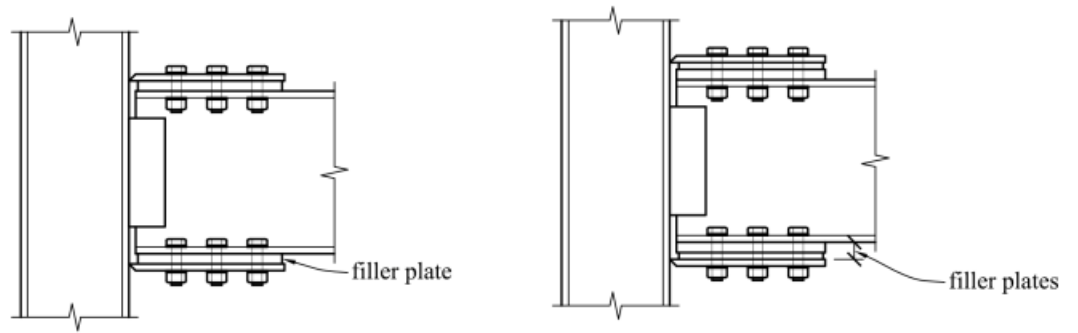


FIGURE 9-7 Clear distance for bolt bearing



a) single filler plate

b) multiple filler plates

FIGURE 9-8 Filler plates

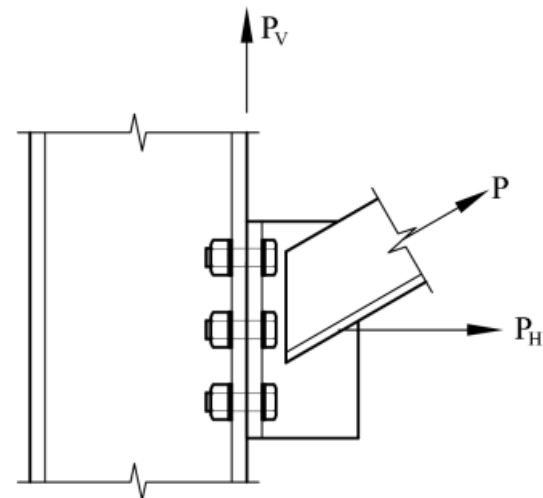


FIGURE 9-9 Connection subjected to combined loading

EXAMPLE 9-2

High-Strength Bolts in a Slip-Critical Connection

Repeat Example 9-1 with ASTM F3125 Grade A325SC bolts with a Class A faying surface.

Solution (LRFD)

Check bolt bearing:

The clear distance between adjacent bolt holes is

$$L_{c1} = 1.5 \text{ in.} - (0.5) \left(\frac{3}{4} + \frac{1}{8} \right) = 1.06 \text{ in.}$$

$$L_{c2} = 3 \text{ in.} - \left(\frac{3}{4} + \frac{1}{8} \right) = 2.13 \text{ in.}$$

For bolt 1,

$$\begin{aligned} \phi R_n &= \phi 1.2 L_c t F_u \leq \phi 2.4 d t F_u \\ &= (0.75)(1.2)(1.06)(0.625)(58 \text{ ksi}) \leq (0.75)(2.4)(0.75)(0.625)(58 \text{ ksi}) \\ &= 34.6 \text{ kips} \leq 48.9 \text{ kips} \end{aligned}$$

Therefore, ϕR_n (bolt 1) = 34.6 kips

For bolt 2,

$$\begin{aligned} \phi R_n &= (0.75)(1.2)(2.13)(0.625)(58 \text{ ksi}) \\ &\leq (0.75)(2.4)(0.75)(0.625)(58 \text{ ksi}) \\ &= 69.5 \text{ kips} \leq 48.9 \text{ kips} \end{aligned}$$

Therefore, ϕR_n (bolt 2) = 48.9 kips

The total strength of the connection in bearing and tearout, considering all the bolts, is

$$\phi R_n = (2)(34.6 \text{ kips}) + (2)(48.9 \text{ kips}) = 167 \text{ kips}$$

$$> P_u = 60 \text{ kips. OK. for bearing and tearout}$$

Check direct shear on the bolts in the slip critical connection:

$$n_s = 1 \text{ (one slip plane)}$$

$$T_b = 28 \text{ kips for } \frac{3}{4} \text{ in. diameter A325 bolt (Table 9-1)}$$

$$h_f = 1 \text{ (no filler plate)}$$

$$D_u = 1.13$$

$$\mu = 0.30 \text{ (Class A faying surface)}$$

$$\phi = 1.0 \text{ (bolt in standard hole in a slip-critical connection)}$$

The slip-critical design shear strength for one bolt is

$$\phi R_n = \phi \mu D_u h_f T_b n_s$$

$$= (1.0)(0.30)(1.13)(1.0)(28)(1)$$

$$= 9.49 \text{ kips/bolt (agrees with AISC, Table 7-3)}$$

The total slip-critical shear strength of the bolt group (with a total of 4 bolts) is

$$\phi R_n = (4)(9.49 \text{ kips}) = 38.0 \text{ kips} < 60 \text{ kips. the connection is not good for shear}$$

The design shear strength of this slip-critical connection is 38 kips. Therefore, the connection is not adequate.

Solution (ASD)

Check bolt bearing:

$$\Omega = 2.0 \text{ (bolt bearing)}$$

The clear distance between adjacent bolt holes is

$$L_{c1} = 1.5 \text{ in.} - (0.5) \left(\frac{3}{4} + \frac{1}{8} \right) = 1.06 \text{ in.}$$

$$L_{c2} = 3 \text{ in.} - \left(\frac{3}{4} + \frac{1}{8} \right) = 2.13 \text{ in.}$$

For bolt 1,

$$\begin{aligned} \frac{R_n}{\Omega} &= \frac{1.2 L_c t F_u}{\Omega} \leq \frac{2.4 d t F_u}{\Omega} \\ &= \frac{(1.2)(1.06 \text{ in.})(0.625 \text{ in.})(58 \text{ ksi})}{2.0} \\ &\leq \frac{(2.4)(0.75)(0.625 \text{ in.})(58 \text{ ksi})}{2.0} \\ &= 23.0 \text{ kips} \leq 32.6 \text{ kips} \end{aligned}$$

$$\text{Therefore, } \frac{R_n}{\Omega} (\text{bolt 1}) = 23.0 \text{ kips.}$$

For bolt 2,

$$\begin{aligned} \frac{R_n}{\Omega} &= \frac{(1.2)(2.13 \text{ in.})(0.625 \text{ in.})(58 \text{ ksi})}{2.0} \\ &\leq \frac{(2.4)(0.75)(0.625 \text{ in.})(58 \text{ ksi})}{2.0} \\ &= 46.3 \text{ kips} \leq 32.6 \text{ kips} \end{aligned}$$

$$\text{Therefore, } \frac{R_n}{\Omega} (\text{bolt 2}) = 32.6 \text{ kips.}$$

The total strength of the connection in bearing and tearout, considering all the bolts, is

$$\frac{R_n}{\Omega} = (2)(23.0 \text{ kips}) + (2)(32.6 \text{ kips}) = 111 \text{ kips}$$
$$> P_S = 40 \text{ kips. OK. for bearing and tearout.}$$

Check direct shear on the bolts in the slip-critical connection:

Ω (standard hole for bolt in slip-critical connection) = 1.5

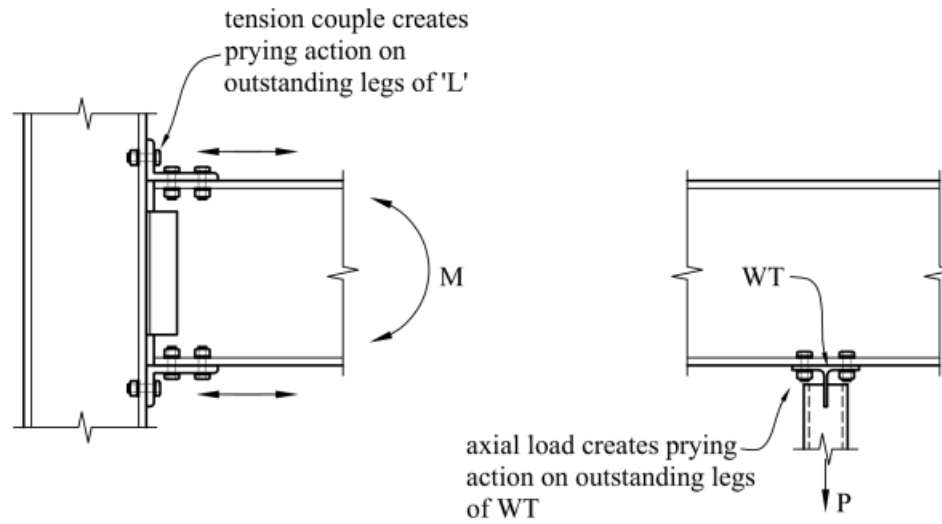
For a single bolt, the available shear strength is

$$\frac{R_n}{\Omega} = \frac{\mu D_u h_{sc} T_b n_s}{\Omega}$$
$$= \frac{(0.30)(1.13)(1.0)(28)(1)}{1.5}$$
$$= 6.33 \text{ kips/bolt (agrees with AISC Table 7-3)}$$

The total shear strength of the bolt group (with a total of 4 bolts) is

$$\frac{R_n}{\Omega} = (4)(6.33 \text{ kips}) = 25.3 \text{ kips} < 40 \text{ kips, the connection not good for shear}$$

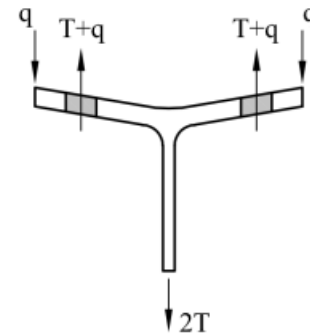
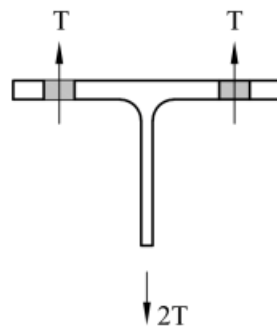
The strength of the connection is the smaller of the allowable bearing strength (111 kips) and the allowable slip-critical shear strength (25.3 kips), yielding a connection strength of 25.3 kips. Therefore, the connection is not adequate to support the service load $P_S = 40$ kips.



Semi-rigid Moment Connection

Hanger Connection

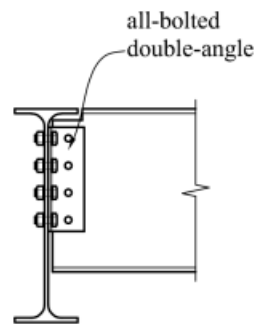
FIGURE 9-26 Connections with prying action



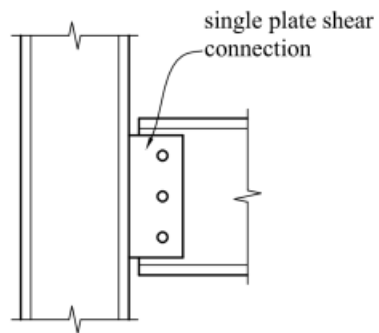
Hanger legs are sufficiently rigid

Hanger legs are not sufficiently rigid

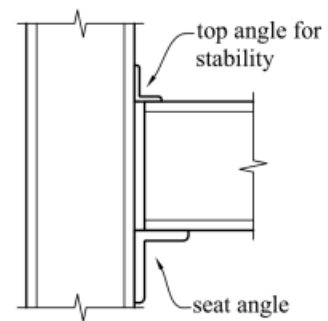
FIGURE 9-27 Prying action on a hanger



a. beam-to-beam



b. beam-to-column



c. beam bearing

FIGURE 9-31 Common types of framed beam connections

EXAMPLE 9-9

All-Bolted Double-Angle Connection

Determine the shear capacity of the connection shown in Figure 9-33 using *AISC*M, Table 10-1. The $W18 \times 35$ beam frames into a $W24 \times 55$ girder. The bolts are $\frac{3}{4}$ in. diameter ASTM F3125 Grade F1852N bolts in standard holes. The angles are ASTM A36 and the beam is ASTM A992, grade 50.

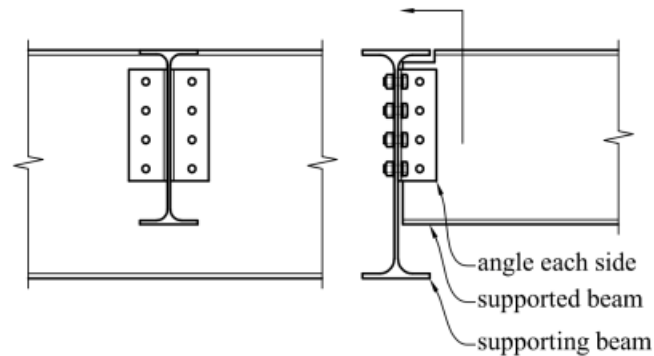


FIGURE 9-32 All bolted double angle connection

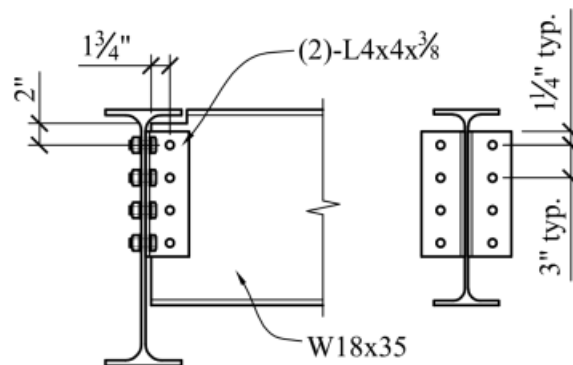


FIGURE 9-33 Connection detail for Example 9-9

Solution

From *AISC*M, Table 10-1, with the number of horizontal rows of bolt, $N = 4$ for $3/4$ -in.-diameter Group A bolts with thread condition “N” and an angle thickness of $3/8$ in., the design strength is found to be 140 kips. The capacity of the bolts bearing on the beam, and the block shear on the beam also must be checked.

With reference to Figure 9-34, the L_{eh} value of $1\frac{3}{4}$ in. will be reduced by $1/4$ in. to allow for fabrication tolerances. The capacity of the bolts bearing on the beam web is

$$L_{e1} = 2 \text{ in.} - 0.5 \left(\frac{3}{4} + \frac{1}{8} \right) = 1.56 \text{ in.}$$

The clear distance between the bolts is:

$$L_{e2} = L_{e3} = L_{e4} = 3 \text{ in.} - \left(\frac{3}{4} + \frac{1}{8} \right) = 2.12 \text{ in.}$$

For W18 $\times$ 35 ASTM A992 Grade 50, $t_w = 0.3$ in.; $F_u = 65$ ksi

Diameter of bolt = $3/4$ in.

For bolt 1 (i.e., the bolt closest to the cope),

$$\begin{aligned} \phi R_n &= \phi 1.2 L_e t F_u \leq \phi 2.4 d t F_u \\ &= (0.75)(1.2)(1.56 \text{ in.})(0.3 \text{ in.})(65 \text{ ksi}) \\ &\leq (0.75)(2.4)(0.75 \text{ in.})(0.3 \text{ in.})(65 \text{ ksi}) \\ &= 27.4 \text{ kips} \leq 26.3 \text{ kips} \end{aligned}$$

Therefore, $\phi R_{n1} = 26.3$ kips.

For bolts 2, 3, and 4,

$$\begin{aligned} \phi R_n &= (0.75)(1.2)(2.12 \text{ in.})(0.3 \text{ in.})(65 \text{ ksi}) \\ &\leq (0.75)(2.4)(0.75 \text{ in.})(0.3 \text{ in.})(65 \text{ ksi}) \\ &= 37.2 \text{ kips} \leq 26.3 \text{ kips} \end{aligned}$$

Therefore, $\phi R_{n2} = \phi R_{n3} = \phi R_{n4} = 26.3$ kips.

The total strength of all the bolts in bearing, is

$$\phi R_n = 26.3 \text{ kips} + (3)(26.3 \text{ kips}) = 105 \text{ kips.}$$

Note that since the web thickness for the W24 $\times$ 55 is 0.395 in., the bolt bearing on the supported W18 $\times$ 35 (with $t_w = 0.3$ in.) will control.

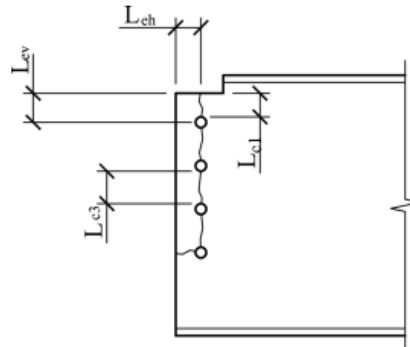


FIGURE 9-34 Bolt bearing and block shear in beam

Block shear in the W18 × 35:

Number of holes on shear plane = 3.5 holes

Number of holes on tension plane = 0.5 hole

$$A_{gv} = [2 \text{ in.} + (3)(3 \text{ in.})](0.3 \text{ in.}) = 3.3 \text{ in.}^2$$

$$A_{nv} = \left[2 \text{ in.} + (3)(3 \text{ in.}) - 3.5 \left(\frac{3}{4} + \frac{1}{8} \right) \right] 0.3 \text{ in.} = 2.38 \text{ in.}^2$$

$$A_{gt} = (L_{eh} - \text{tolerance}) = (1.75 \text{ in.} - 0.25 \text{ in.})(0.3 \text{ in.}) = 0.45 \text{ in.}^2$$

$$\begin{aligned} A_{nt} &= \left[(L_{eh} - \text{tolerance}) - 0.5 \left(\frac{3}{4} + \frac{1}{8} \right) \right] (0.3 \text{ in.}) \\ &= \left[(1.75 \text{ in.} - 0.25 \text{ in.}) - 0.5 \left(\frac{3}{4} + \frac{1}{8} \right) \right] (0.3 \text{ in.}) = 0.318 \text{ in.}^2 \end{aligned}$$

Using equation (4-11a), to determine the block shear strength,

$$\begin{aligned} \phi R_n &= \phi (0.6 F_u A_{nv} + U_{bs} F_u A_{nt}) \leq \phi (0.6 F_y A_{gv} + U_{bs} F_u A_{nt}) \\ &= 0.75 [(0.6)(65)(2.38) + (1.0)(65)(0.318)] \\ &\leq 0.75 [(0.6)(50)(3.3) + (1.0)(65)(0.318)] \\ &= 85.1 \text{ kips} \leq 89.8 \text{ kips} \end{aligned}$$

Therefore, $\phi R_n = 85.1 \text{ kips} < 105 \text{ kips}$;

That is, the block shear capacity of the supported W18 × 35 beam (85.1 kips) controls over bolt bearing or tearout (105 kips) and the shear capacity of the double angle connection elements (140 kips).

Therefore, the design shear strength of this connection, $\phi R_n = 85.1 \text{ kips}$.

Bolted/Welded Shear End-Plate Connections

A bolted/welded shear end-plate connection is shown in Figure 9-35. For this type of connection, the failure modes that must be considered are shear in the bolts, bearing of the bolts, direct shear and block shear in the end plate, and shear in the weld connecting the supported beam to the end plate (welds are covered in Chapter 10). *AISC*M, Table 10-4, should be used for these types of connections.

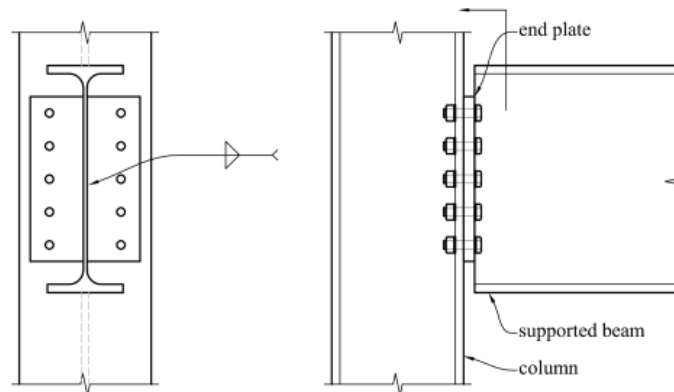
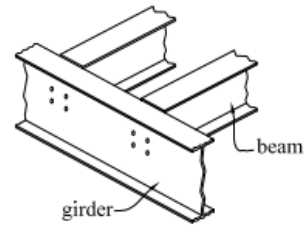
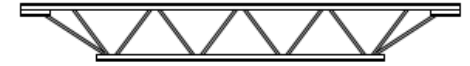


FIGURE 9-35 Bolted/welded shear end-plate connections

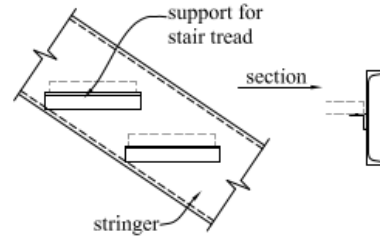
Flexural Members



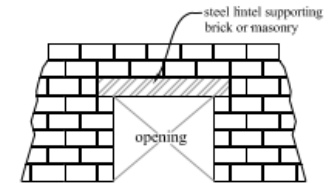
a. floor beams and girders



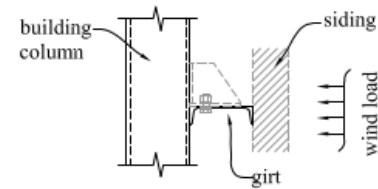
b. open-web steel joist



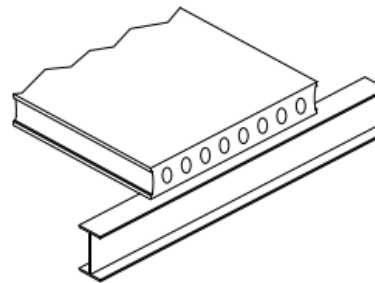
c. stringer



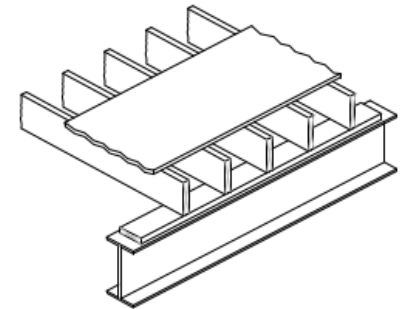
d. loose lintel



e. girt



f. beam supporting concrete plank



g. beam supporting wood framing

FIGURE 6-1 Common beam members

$$M_y = F_y S,$$

where

f_b = Maximum bending stress,

 $M_y =$ Yield moment, $F_y =$ Yield stress, $\dot{M}$ = Bending moment,

c = Distance from the neutral axis to the extreme fiber,

I = Moment of inertia, and

S = Elastic section modulus.

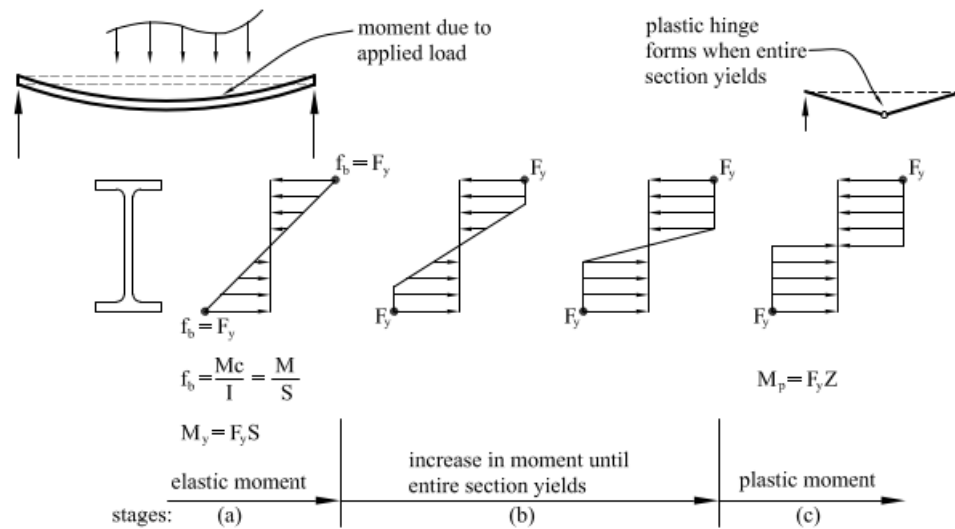
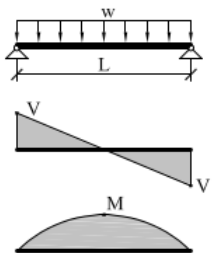


FIGURE 6-2 Stress distribution in bending members

TABLE 6-1 Summary of Shear, Moment, and Deflection Formulas

Loading condition	Loading diagram	Maximum shear	Maximum moment	Maximum deflection
Uniformly loaded simple span		$V = \frac{wL}{2}$	$M = \frac{wL^2}{8}$	$\Delta = \frac{5wL^4}{384EI}$

(continued)
(continued)

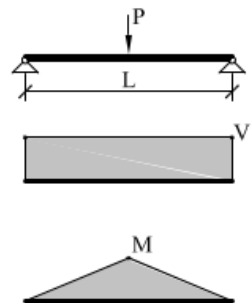
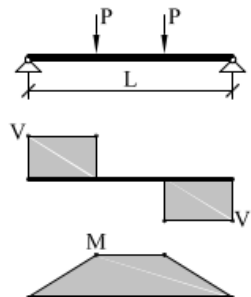
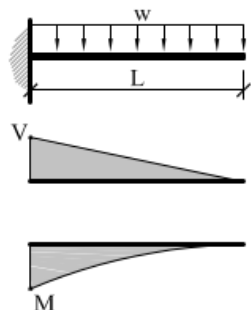
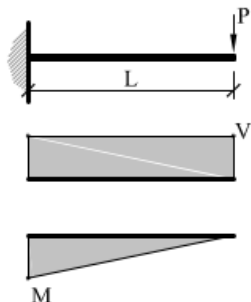
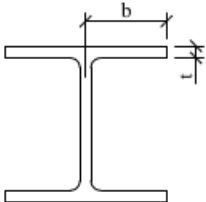
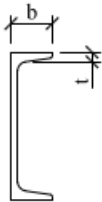
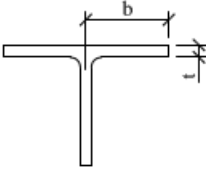
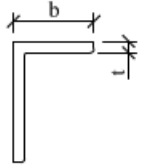
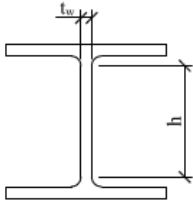
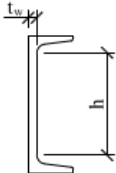
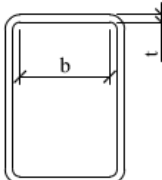
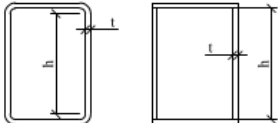
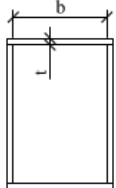
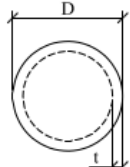
Loading condition	Loading diagram	Maximum shear	Maximum moment	Maximum deflection
Concentrated load at midspan		$V = \frac{P}{2}$	$M = \frac{PL}{4}$	$\Delta = \frac{PL^3}{48EI}$
Concentrated loads at 1/3 points		$V = P$	$M = \frac{PL}{3}$	$\Delta = \frac{PL^3}{28EI}$
Uniformly loaded, cantilever		$V = wL$	$M = \frac{wL^2}{2}$	$\Delta = \frac{wL^4}{8EI}$
Concentrated load at end of cantilever		$V = P$	$M = PL$	$\Delta = \frac{PL^3}{3EI}$

TABLE 6-2 Limiting Width–Thickness Ratios for Flexural Elements

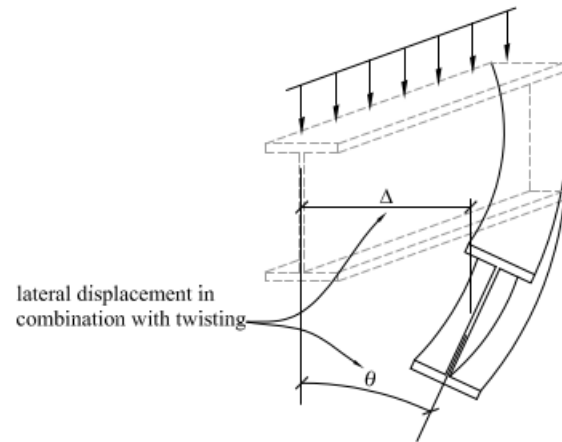
Unstiffened in insert	Description	Limiting width–thickness ratio		Details
		λ_{pf} λ_{pw} (web)(compact)	λ_{rf} λ_{rw} (web)(noncompact)	
	Flanges of I-shaped sections (Case 10)	$\frac{b}{t} \leq 0.38 \sqrt{\frac{E}{F_y}}$	$\frac{b}{t} \leq 1.0 \sqrt{\frac{E}{F_y}}$	
	Flanges of C-shapes (Case 10)			
	Flanges of WT-shapes (Case 10)			
	Outstanding legs of single angles (Case 12)			

(continued)

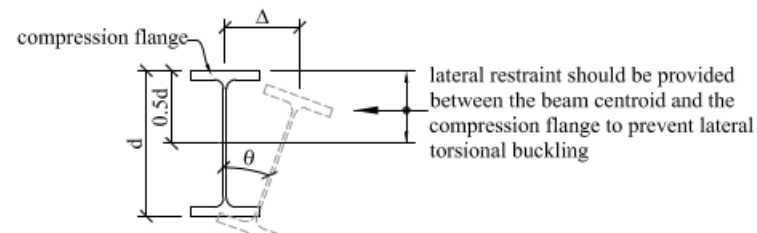
(continued)

Stiffened	Description	Limiting width-thickness ratio		Details
		λ_p λ_{pf} (flange) λ_{pw} (web) (compact)	λ_r λ_{rf} (flange) λ_{rw} (web) (noncompact)	
	Webs of I-shaped sections (Case 15)	$\frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}}$	$\frac{h}{t_w} \leq 5.70 \sqrt{\frac{E}{F_y}}$	
	Webs of C-shapes (Case 15)			
	Flanges of rectangular HSS (Case 17)	$\frac{b}{t} \leq 1.12 \sqrt{\frac{E}{F_y}}$	$\frac{b}{t} \leq 1.40 \sqrt{\frac{E}{F_y}}$	
	Webs of rectangular HSS and Box sections (Case 19)	$\frac{h}{t} \leq 2.42 \sqrt{\frac{E}{F_y}}$	$\frac{h}{t} \leq 5.70 \sqrt{\frac{E}{F_y}}$	
	Flanges of Box sections (Case 21)	$\frac{b}{t} \leq 1.12 \sqrt{\frac{E}{F_y}}$	$\frac{b}{t} \leq 1.49 \sqrt{\frac{E}{F_y}}$	
	Round HSS or pipes (Case 20)	$\frac{D}{t} \leq 0.07 \frac{E}{F_y}$	$\frac{D}{t} \leq 0.31 \frac{E}{F_y}$	

Source: Adapted from Table B4.1b of the AISC.[1]. Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved.



a. lateral torsional buckling behavior



b. lateral torsional buckling restraint

FIGURE 6-3 Lateral torsional buckling

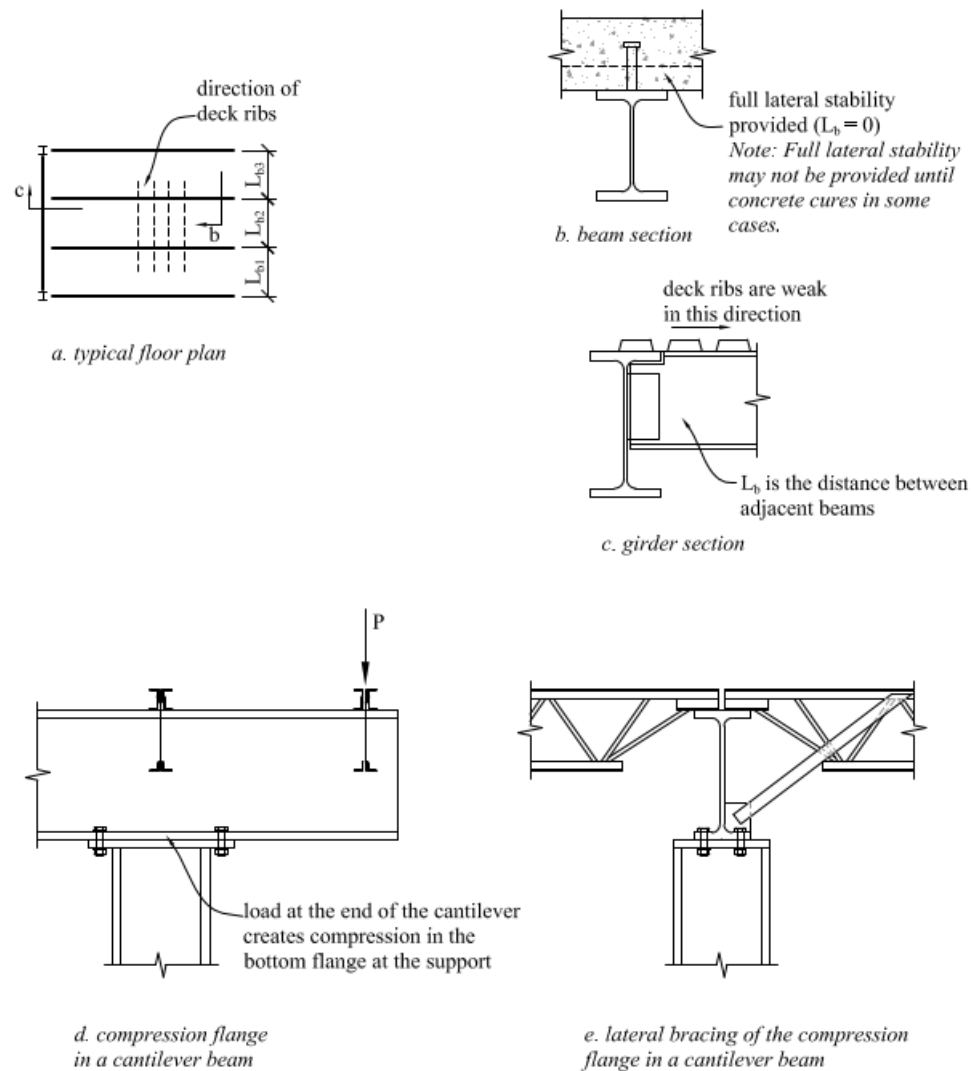
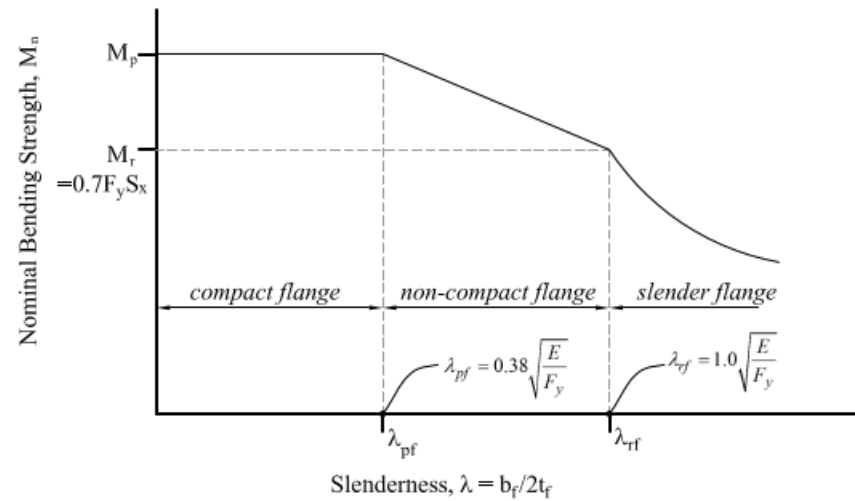


FIGURE 6-4 Lateral bracing details

$$M_u \leq \phi_b M_n \quad \text{for LRFD}$$

$$M_s \leq \frac{M_n}{\Omega_b} \quad \text{for ASD}$$



- a) Nominal moment strength variation with flange slenderness for I shape
(Adapted from AISCM Figure C-F1.1)

EXAMPLE 6-3

Bending Strength of a Wide Flange Beam

Determine the available bending strength (M_n/Ω_b) and the design bending strength ($\phi_b M_n$), for a W14 $\times$ 74 flexural member of ASTM A992, grade 50 steel, assuming

1. Continuous lateral support;
2. Unbraced length = 15 ft., $C_b = 1.0$; and
3. Unbraced length = 15 ft., $C_b = 1.30$.

Solution

From part 1 of the *AISC*, the section properties for W14 $\times$ 74 are

$$Z_x = 126 \text{ in.}^3$$

$$J = 3.87 \text{ in.}^4$$

$$r_{ts} = 2.83 \text{ in.}$$

$$S_x = 112 \text{ in.}^3$$

$$h_o = 13.4 \text{ in.}$$

$$r_y = 2.48 \text{ in.}$$

Check the compact section criteria for the W14 $\times$ 74 flexural member:

Flange:

The actual flange width-to-thickness ratio, $\frac{b_f}{2t_f}$, for W14 $\times$ 74 is 6.41.

The maximum flange width-to-thickness ratio to ensure a compact flange is

$$\lambda_{pf} = 0.38 \sqrt{\frac{E}{F_y}} = 0.38 \sqrt{\frac{29,000}{50}} = 9.15 > 6.41,$$

$\therefore$ the W14 $\times$ 74 flange is compact.

Web:

The actual web depth-to-thickness ratio, $\frac{h}{t_w}$, for W14 $\times$ 74 is 25.4.

The maximum web depth-to-thickness ratio to ensure a compact web is

$$\lambda_{pw} = 3.76 \sqrt{\frac{E}{F_y}} = 3.76 \sqrt{\frac{29,000}{50}} = 90.5 > 25.4,$$

$\therefore$ the W14 $\times$ 74 web is compact.

1. Continuous lateral support, $L_b = 0$ (upper linear zone in Figure 6-5)

$$\begin{aligned}
 M_n &= M_p = F_y Z_x \\
 &= (50)(126) = 6300 \text{ in. kips} = 525 \text{ ft.-kips} \\
 \phi_b M_n &= (0.9)(525) = \mathbf{473 \text{ ft.-kips}} \text{ for LRFD} \\
 \frac{M_n}{\Omega_b} &= \frac{525}{1.67} = \mathbf{314 \text{ ft.-kips}} \text{ for ASD}
 \end{aligned}$$

2. $L_b = 15 \text{ ft.}$ and $C_b = 1.0$

$$\begin{aligned}
 L_p &= 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.48) \sqrt{\frac{29,000}{50}} = 105.1 \text{ in.} = 8.76 \text{ ft.} < L_b = 15 \text{ ft.} \\
 L_r &= 1.95 r_{ts} \frac{E}{0.7 F_y} \sqrt{\left(\frac{Jc}{S_x h_o}\right)} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7 F_y}{E} \frac{S_x h_o}{Jc}\right)^2}} \\
 &= 1.95(2.83) \frac{29,000}{0.7(50)} \sqrt{\left(\frac{(3.87)(1.0)}{(112)(13.4)}\right)} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7(50)}{29,000} \frac{(112)(13.4)}{(3.87)(1.0)}\right)^2}} \\
 &= 372.5 \text{ in.} = 31.0 \text{ ft.} > L_b = 15 \text{ ft.}
 \end{aligned}$$

Note that $c = 1.0$ for I-shapes.

Since $L_p < L_b < L_r$, the nominal moment strength is found from equation (6-10):

$$\begin{aligned}
 M_n &= C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \\
 &= (1.0) \left[6300 - (6300 - (0.7)(50)(112)) \left(\frac{15 - 8.76}{31.0 - 8.76} \right) \right] \leq 6300 \\
 M_n &= 5632 \text{ in.-kips} = 469 \text{ ft.-kips}
 \end{aligned}$$

$$\phi_b M_n = (0.9)(469) = \mathbf{422 \text{ ft.-kips}} \text{ for LRFD}$$

$$\frac{M_n}{\Omega_b} = \frac{469}{1.67} = \mathbf{280 \text{ ft.-kips}} \quad \text{for ASD}$$

3. $L_b = 15 \text{ ft.}$ and $C_b = 1.3$

$$\begin{aligned}
 M_n &= C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \\
 &= 1.3 \left[6300 - (6300 - (0.7)(50)(112)) \left(\frac{15 - 8.76}{31.0 - 8.76} \right) \right] \leq 6300
 \end{aligned}$$

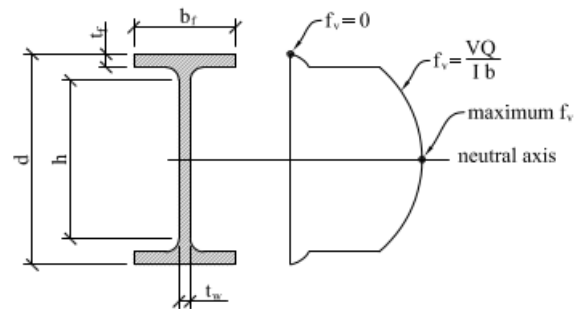
$$M_n = 7321 \text{ in.-kips} > 6300 \text{ in.-kips, } \therefore \text{ flexural yielding controls.}$$

$$M_n = M_p = 6300 \text{ in.-kips} = 525 \text{ ft.-kips}$$

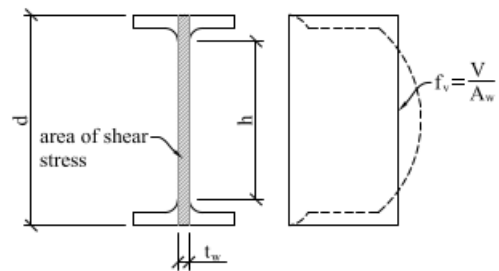
$$\phi_b M_n = (0.9)(525) = \mathbf{473 \text{ ft.-kips}} \quad \text{for LRFD}$$

$$\frac{M_n}{\Omega_b} = \frac{525}{1.67} = \mathbf{314 \text{ ft.-kips}} \quad \text{for ASD}$$

DESIGN OF FLEXURAL MEMBERS FOR SHEAR



a. actual shear stress



b. approximated shear stress

FIGURE 6-7 (a-b) Shear in a beam

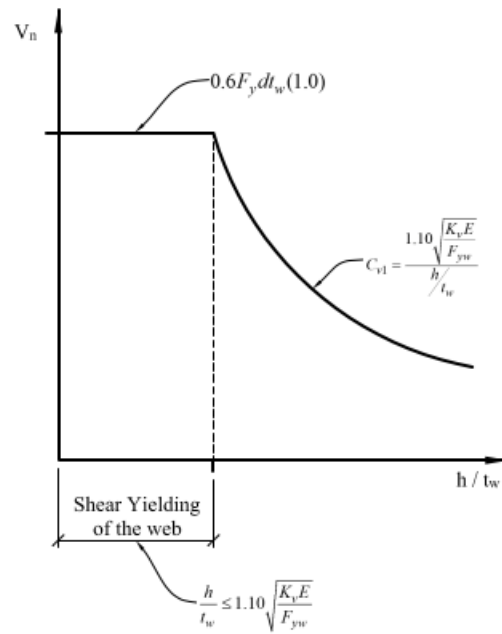


FIGURE 6-7c Shear strength variation in a flexural member

EXAMPLE 6-5

Shear Strength of a Steel Shape

Determine the design shear strength of the following sections, using $F_y = 50$ ksi for the W-shapes and $F_y = 36$ ksi for the C-shape:

1. W16 $\times$ 26, and
2. W18 $\times$ 50.
3. C12 $\times$ 20.7

Solution

1. W16 $\times$ 26

From *AISC*M, Table 1-1,

$$t_w = 0.25 \text{ in.}$$

$$d = 15.7 \text{ in.}$$

$$\frac{h}{t_w} = 56.8$$

$$\frac{h}{t_w} \leq 2.24 \sqrt{\frac{E}{F_y}}$$

$$2.24 \sqrt{\frac{29,000}{50}} = 54.0 < h / t_w = 56.8, \therefore \phi_v = 0.9.$$

Since $\frac{h}{t_w} = 56.8 > 2.24 \sqrt{\frac{E}{F_y}} = 54$, therefore, $\phi_v = 0.9$

Determine C_{v1} :

$$\text{If } \frac{h}{t_w} \leq 1.10 \sqrt{\frac{k_v E}{F_y}}, C_{v1} = 1.0$$

$$1.10 \sqrt{\frac{(5.34)(29,000)}{50}} = 61.2 > h/t_w = 56.8, \therefore C_{v1} = 1.0.$$

For $W16 \times 26$, $\phi_v = 0.9$ and $\Omega_v = 1.67$.

From equation (6-18),

$$\phi_v V_n = \phi_v 0.6 F_y A_w C_{v1} = (0.9)(0.6)(50)(0.25)(15.7)(1.0) = \mathbf{106 \text{ kips. for LRFD}}$$

From equation (6-19),

$$\frac{V_n}{\Omega_v} = \frac{0.6 F_y A_w C_{v1}}{\Omega_v} = \frac{(0.6)(50)(0.25)(15.7)(1.0)}{1.67} = \mathbf{70.5 \text{ kips. for ASD}}$$

2. W18 × 50

From *AISC*M, Table 1-1,

$$t_w = 0.355 \text{ in.}$$

$$d = 18.0 \text{ in.}$$

Since $W18 \times 50$ meets the requirement of equation (6-20a), therefore, $C_{v1} = 1.0$, $\Omega_v = 1.5$ and $\phi_v = 1.0$

From equation (6-18),

$$\phi_v V_n = \phi_v 0.6 F_y A_w C_{v1} = (1.0)(0.6)(50)(0.355)(18.0)(1.0) = \mathbf{192 \text{ kips. for LRFD}}$$

From equation (6-19),

$$\frac{V_n}{\Omega_v} = \frac{0.6 F_y A_w C_{v1}}{\Omega_v} = \frac{(0.6)(50)(0.355)(18.0)(1.0)}{1.5} = \mathbf{128 \text{ kips. for ASD}}$$

3. C12 × 20.7

From *AISC*M, Table 1-5,

$$t_w = 0.282 \text{ in.}$$

$$d = 12 \text{ in.}$$

For a $C12 \times 20.7$ which is a channel, $C_{v1} = 1.0$, $\Omega_v = 1.67$ and $\phi_v = 0.9$ as discussed previously.

From equation (6-18),

$$\phi_v V_n = \phi_v 0.6 F_y A_w C_{v1} = (0.9)(0.6)(36)(12)(0.282)(1.0) = \mathbf{65.8 \text{ kips. for LRFD}}$$

From equation (6-19),

$$\frac{V_n}{\Omega} = \frac{0.6 F_y A_w C_{v1}}{\Omega} = \frac{(0.6)(36)(0.282)(12)(1.0)}{1.67} = \mathbf{43.8 \text{ kips. for ASD}}$$

TABLE 6-4 Deflection Limits for Beams

Member description	Live load	Snow load or wind load	Dead plus live load
Roof members			
Supporting plaster ceiling	$L/360$	$L/360$	$L/240$
Supporting nonplaster ceiling	$L/240$	$L/240$	$L/180$
Not supporting ceiling	$L/180$	$L/180$	$L/120$
Floor Members	$L/360$	N/A	$L/240$

BEAM DESIGN PROCEDURE

The typical design procedure for beams involves selecting a member that has adequate strength in bending and adequate stiffness for serviceability. Shear typically does not control, but it should be checked. The beam or girder design process using the LRFD method is as follows:

1. Determine the service and factored loads on the beam. Service loads are used for deflection calculations and factored loads are used for strength design. The weight of the beam is unknown at this stage, but the self-weight can be initially estimated and is usually comparatively small enough that a change in its value will not affect the design.
2. Determine the factored shear and moments on the beam.
3. Select a shape that satisfies strength and deflection criteria. One of the following methods can be used:
 - a. For the steel sections listed in the AISC beam design tables, select the most economical beam to support the factored moment. Then check deflection and shear for the selected shape.
 - b. Determine the required minimum moment of inertia using Table 6-5. Select the most economical shape based on the calculated moment of inertia and check the selected section for bending and shear.
 - c. For shapes not listed in the AISC beam design tables, an initial size must be assumed. An estimate of the available bending strength can be made for an initial beam selection; then check shear and deflection. A more accurate method might be to follow the procedure in step 3b.
4. Check floor vibrations (see Chapter 12).

The beam or girder design process using the ASD method would follow a similar approach as the preceding, but the ASD load combinations in Section 2-3 are used.

EXAMPLE 6-9

Floor Beam and Girder Design

For the floor plan shown in Figure 6-8, design members B1 and G1 for bending, shear, and deflection. Compare deflections with $L/240$ for total loads and with $L/360$ for live loads. The steel is ASTM A992, grade 50; assume that $C_b = 1.0$ for bending. The dead load (including the beam weight) is assumed to be 85 psf and the live load 150 psf. Assume that the floor deck provides full lateral stability to the top flange of B1. Ignore live load reduction. Use the design tables in the *AISC* where appropriate.

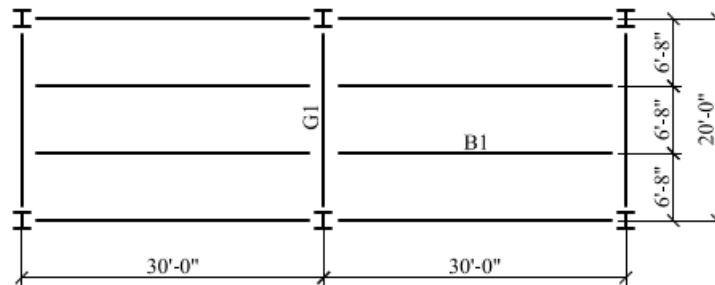


FIGURE 6-8 Floor plan for Example 6-9

Solution

Since the dead load is more than half of the live load, the total load deflection of $L/240$ will control.

Summary of Loads (see Figure 6-9):

$$L = 150 \text{ psf}$$

$$P_s = 85 + 150 = 235 \text{ psf} = 0.235 \text{ ksf (total service load)}$$

$$P_u = (1.2)(85) + (1.6)(150) = 342 \text{ psf} = 0.342 \text{ ksf (total factored load)}$$

Beam B1

$$\text{Tributary width} = 6 - 8'' = 6.67 \text{ ft.}$$

$$\text{Span of the beam, } L = 30 \text{ ft.}$$

$$\text{Tributary area} = (6.67')(30') = 200 \text{ ft.}^2$$

$$w_L = (6.67)(0.150) = 1.0 \text{ kips/ft}$$

$$w_s = w_{D+L} = (6.67)(0.235) = 1.57 \text{ kips/ft.}$$

$$w_u = (6.67)(0.342) = 2.28 \text{ kips/ft.}$$

$$V_u = \frac{w_u L}{2} = \frac{(2.28)(30)}{2} = 34.2 \text{ kips}$$

$$\text{Factored beam reaction} = 34.2 \text{ kips}$$

$$M_u = \frac{w_u L^2}{8} = \frac{(2.28)(30)^2}{8} = 257 \text{ ft.-kips}$$

The loading diagrams for B-1 and G-1 are shown in Figure 6-9.

Girder G1

$$\text{Span of girder} = 20 \text{ ft.}$$

$$P_L = (200)(0.150) = 30 \text{ kips}$$

$$P_s = P_{D+L} = \left(\frac{200}{2} + \frac{200}{2} \right) (0.235) = 47 \text{ kips}$$

$$P_u = \left(\frac{200}{2} + \frac{200}{2} \right) (0.342) = 68.4 \text{ kips}$$

$$V_u = P_u = 68.4 \text{ kips}$$

$$M_u = \frac{P_u L}{3} = \frac{(68.4)(20)}{3} = 456 \text{ ft.-kips}$$

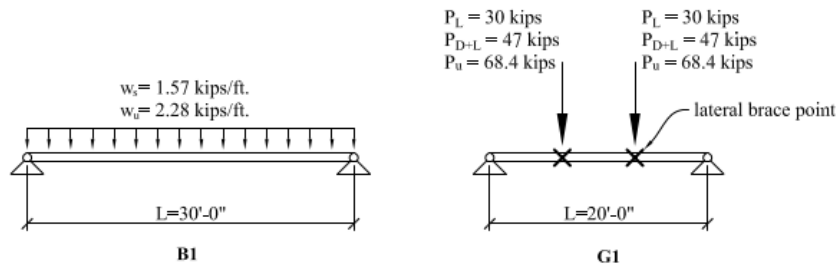


FIGURE 6-9 Loading for B1 and G1

1. Design of Beam B1

We will use the *AISC* beam design charts and the required minimum moment of inertia method to select a beam size.

$M_u = 257 \text{ ft.-kips}$; and the unbraced length, $L_b = 0$, since the compression flange of the beam is fully laterally braced by the floor deck.

From *AISC* Table 3-10, a $W16 \times 40$ is selected as the most economical size for bending, with $\phi_b M_n = 274 \text{ ft.-kips}$. However, note that a $W18 \times 40$ has the same beam weight and, therefore, virtually the same cost, but provides more strength ($\phi_b M_n = 294 \text{ ft.-kips}$) and stiffness ($I = 612 \text{ in.}^4$ versus $I = 518 \text{ in.}^4$). Therefore, a $W18 \times 40$ is initially selected.

From Table 6-5 (total load controls deflection),

$$I_{\text{required}} = \frac{w_{D+L} L^3}{64.44} = \frac{(1.57)(30)^3}{64.44} = 658 \text{ in.}^4$$

The required moment of inertia is greater than the moment of inertia of the $W18 \times 40$, which is 612 in.^4 ; therefore, a new beam size needs to be selected.

From *AISC* Table 1-1,

$$W16 \times 50, I = 659 \text{ in.}^4$$

$$W18 \times 46, I = 712 \text{ in.}^4$$

$$W21 \times 44, I = 843 \text{ in.}^4 \leftarrow \text{Select}$$

$$W24 \times 55, I = 1350 \text{ in.}^4$$

The $W21 \times 44$ has the lightest weight of all the sections above, so this beam size is selected. Alternatively, from *AISC* Table 3-3, a $W21 \times 44$ is found to be the lightest section with a moment of inertia greater than 658 in.^4 . Checking the moment capacity with *AISC* Table 3-10, we find that $\phi_b M_n = 358 \text{ ft.-kips} > M_u = 257 \text{ ft.-kips}$.

Checking shear, note that a $W21 \times 44$ ($t_w = 0.35 \text{ in.}$; $d = 20.7 \text{ in.}$) does not have a slender web; therefore, the design shear strength is determined from equation (6-18), with $C_{v1} = 1.0$ and $\phi_v = 1.0$:

$$\phi_v V_n = \phi_v 0.6 F_y A_w C_{v1} = (1.0)(0.6)(50)(0.35)(20.7)(1.0) = 217 \text{ kips} > V_u = 34.2 \text{ kips} \quad \text{OK.}$$

Alternatively, the shear strength can be found from *AISC* Table 3-6 ($\phi_v V_n = 217 \text{ kips}$, same as above).

A $W21 \times 44$ is selected for member B1.

2. Design of Girder G1

We will use the *AISC* beam design charts and the required moment of inertia method to select a girder size. From Figure 6-9, the maximum factored moment in the girder is calculated as

$$M_u = 456 \text{ ft.-kips and the unbraced length, } L_b = 6.67 \text{ ft.}$$

That is, the girders are assumed to be laterally braced only at the beam locations.

From *AISC* Table 3-10, a $W24 \times 55$ is selected as the most economical size for bending, with $\phi_b M_n = 460 \text{ ft.-kips}$.

Check the total load deflection of the girder using the total unfactored concentrated dead plus live loads, $P_{D+L} = 47 \text{ kips}$. The deflection equation for a girder with two equal and symmetrically located concentrated loads [1] is:

$$\Delta_{D+L} = \frac{P_{D+L} L^3}{28EI} = \frac{(47)[(20)(12)]^3}{(28)(29,000)(1350)} = 0.593 \text{ in.} < \frac{L}{240} = \frac{(20)(12)}{240} = 1 \text{ in. OK.}$$

As an alternate approximate check, use equation (6-23) to determine the deflection:

The total maximum unfactored moment due to $P_{D+L} = 47$ kips is

$$M_{D+L} = \frac{P_{D+L}L}{3} = \frac{(47)(20)}{3} = 313.3 \text{ ft.-kips}$$

With beam span, $L = 20$ ft, the deflection in inches is computed from equation (6-23) as

$$\Delta_{D+L} = \frac{M_{D+L}L^2}{(161.1)I} = \frac{(313.3)(20)^2}{(161.1)(1350)} = 0.589 \text{ in.}$$

Recall that equation (6-23) was developed for the uniform load case, but the results for the concentrated loads at $1/3$ points is close to the actual value of $\Delta = 0.593$ in. (less than a 3% difference). Therefore, equations (6-24) and (6-25), which are also expressed as a function of the moment in a beam, could also have reasonably been used for a quick size selection based on the required stiffness or moment of inertia.

Check Shear:

From Figure 6-9, the maximum factored shear is calculated as 68.4 kips. Note that from Section 6-5, the design shear strength is determined from equation (6-18), with $C_{v1} = 1.0$ and $\phi_v = 0.9$ (recall that a $W24 \times 55$ ($t_w = 0.395$ in., $d = 23.6$ in.) does not meet the web depth-to-thickness ratio from equation (6-20)):

$$\phi_v V_n = \phi_v 0.6F_y A_w C_{v1} = (0.9)(0.6)(50)(50)(0.395)(23.6)(1.0) = 251 \text{ kips} > V_u = 68.4 \text{ kips OK.}$$

Alternatively, the shear strength can be found from *AISC* Table 3-6 ($\phi_v V_n = 251$ kips, same as above).

A $W24 \times 55$ is selected for member G1.